

# ANNUAL CONTROLLED FUSION THEORY CONFERENCE

MAY 4-6, 1977

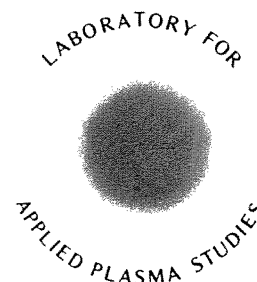
SAN DIEGO, CALIFORNIA

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PROCEEDINGS OF THE ANNUAL CONTROLLED  
FUSION THEORY CONFERENCE

MAY 4-6, 1977

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SAN DIEGO, CALIFORNIA

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## GENERAL INFORMATION

### 1977 ANNUAL CONTROLLED FUSION THEORY CONFERENCE

*The morning sessions will be held in the Islands Room; please check your schedule for starting times. Coffee and other refreshments will be available during the morning and afternoon sessions. Afternoon poster sessions will be held in the hallway off the mezzanine.*

*Please note that, because of the large number of abstracts submitted, it has been necessary to hold poster sessions in two time periods. It is hoped that the afternoon change-over will not be too confusing.*

*Dr. Robert Price of ERDA will give a special talk at 5:00 PM, Thursday, May 5th in the Islands Room. The cocktail party will be held in the same room beginning at 6:30 PM.*

*Registration will be open in the lobby of the Islandia Hyatt House starting at 4:00 PM, May 3, 1977. During the conference, rooms will be available for conversation and study.*

*Please check the bulletin board and the registration desk for tour and restaurant information.*

## SCHEDULE

### 1977 ANNUAL CONTROLLED FUSION THEORY CONFERENCE

#### WEDNESDAY, MAY 4

|                |                                           |
|----------------|-------------------------------------------|
| 8:30 AM        | Welcome, G. Guest and N. Kra11            |
| 8:40 AM        | T. K. Chu - <i>Linear Magnetic Fusion</i> |
| 10:20-10:40 AM | Coffee Break                              |
| 12:30 PM       | LUNCH                                     |
| 1:45-3:00 PM   | Poster Sessions A and B                   |
|                | Coffee Served 2:15-4:15 PM                |
| 3:30-4:45 PM   | Poster Sessions C and D                   |

#### THURSDAY, MAY 5

|                |                                         |
|----------------|-----------------------------------------|
| 8:40 AM        | T. H. Dupree - <i>Strong Turbulence</i> |
| 10:20-10:40 AM | Coffee Break                            |
| 12:30          | LUNCH                                   |
| 1:45-3:00 PM   | Poster Sessions E and F                 |
|                | Coffee Served 2:15-4:15 PM              |
| 3:30-4:45 PM   | Poster Sessions G and H                 |
| 5:00-5:40 PM   | R. Price - <i>MFE Theory Program</i>    |
| 6:30-8:30 PM   | Cocktail Party                          |

#### FRIDAY, MAY 6

|                |                                    |
|----------------|------------------------------------|
| 8:40 AM        | D. Baker - <i>Toroidal Z-Pinch</i> |
| 10:20-10:40 AM | Coffee Break                       |
| 12:30 AM       | LUNCH                              |
| 1:45-3:00 PM   | Poster Session I and J             |
|                | Coffee Served 2:00-3:00 PM         |

## PROGRAM

### 1977 ANNUAL CONTROLLED FUSION THEORY CONFERENCE

WEDNESDAY, MAY 4

- 8:30 AM WELCOME { Gareth Guest, General Atomic Company  
Nick Krall, Science Applications, Inc.
- PLENARY {  
SESSION} W. Tang and D. Dobrott, Chairman
- PA1 } 8:40 AM T. K. Chu, *"Linear Magnetic Fusion"*  
PA2 }
- PA3 B. V. Waddell, J. D. Callen, B. Carreras, H. R. Hicks,  
J. Holmes, and G. L. Jahns, *"Tearing Modes in Tokamaks"*
- PA4 Liu Chen, P. H. Rutherford, and W. M. Tank, *"Drift-  
Tearing Instabilities Due to Trapped Electron"*
- PA5 J. F. Drake and Y. C. Lee, *"Kinetic Theory of Tearing  
Instabilities"*
- PA6 K. Lackner and F. Karger, *"Stabilization of Disruptive  
Instability"*
- 10:20-10:40 AM COFFEE BREAK
- PA7 P. C. Liewer, A. W. Allen, F. L. Cochran, and G. C.  
Goldenbaum, *"Theoretical and Experimental Studies of  
MHD Stability in a Finite-Beta Rectangular Cross  
Section Toroidal Plasma Device"*
- PA8 M. S. Chance, J. M. Greene, R. C. Grimm, Y. Y. Hsieh,  
J. L. Johnson, J. Manickam, M. Okabayashi, A. M. M.  
Todd, and D. E. Voss, *"Tokamak Stability Studies Using  
the PEST Code"*
- PA9 { D. Dobrott and D. Lulue, *"Localized Ballooning Modes in  
Noncircular Cross-Sectioned Tokamaks"*  
A. H. Glasser, J. M. Greene, M. S. Chance, and E. A.  
Frieman, *"Ballooning Modes in Axisymmetric Toroidal  
Plasmas"*
- PA10 D. C. Barnes, J. U. Brackbill, and W. Schneider,  
*"Advances in Scyllac Equilibrium Theory"*

POSTER SESSION A  
(1:45-3:00 PM)

- A1 D. V. Anderson, M. E. Rensink, and W. C. Turner, "*Equilibrium Models for Field Reversal for 2XIIB Parameters*"
- A2 S. P. Auerbach and A. Boozer, "*Classical Diffusion at an X Point*"
- A3 Glenn Bateman and Y-K. M. Peng, "*Large Scale MHD Stability of FCT Tokamaks*"
- A4 B. Carreras and B. V. Waddell, "*Analytic Treatment of the Interaction of Tearing Modes in Tokamaks*"
- A5 Liu Chen, J. Hsu, P. K. Kaw, and P. H. Rutherford, "*Magnetic Tearing Due to Finite- $\beta$  Modified Drift Waves*"
- A6 C. H. Finan III and J. Killeen, "*3-D Nonlinear Evolution of the Internal Kink Mode in a Resistive Plasma*"
- A7 H. Grad and P. N. Hu, "*Classical Diffusion and Energy Transport: Equations and Numerical Algorithms*"
- A8 B. J. Green and H. P. Zehrfeld, "*Local, Ideal and Resistive Stability in Tokamaks with Noncircular Cross Section*"
- A9 F. J. Helton and Reagan Moore, "*Stability Results on Racetrack Equilibria*"
- A10 H. R. Hicks, B. V. Waddell, B. Carreras, and J. Holmes, "*Nonlinear Evolution of the Reduced Set of Three-Dimensional Resistive MHD Equations for Tokamaks*"
- A11 H. C. Lui, "*MHD Stability of High Beta Tokamak Equilibria with Force Free Currents*"
- A12 J. Manickam and B. McCann, "*PEST Documentation*"
- A13 C. Mercier, "*Stationary Axisymmetric States with Diffusion and Heating of Tokamak Configurations*"
- A14 D. A. Monticello, M. N. Rosenbluth, R. B. White, and B. V. Waddell, "*Nonlinear Drift Tearing Modes*"
- A15 Y-K. M. Peng and J. A. Holmes, "*Time Evolution of Flux-Conserving Equilibria with Noncircular Cross Sections in a D-T Tokamak*"

- A16 Michael F. Reusch, *"Non-Equilibrium Linear MHD Stability"*
- A17 D. Schnack and J. Killeen, *"Linear and Non-Linear Numerical Studies of Resistive Instabilities in Cylindrical Geometry"*
- A18 N. Sharky, H. C. Lui, and C. K. Chu, *"The Evolution to Equilibrium of Helically Symmetric Theta Pinches"*
- A19 George Vahala, Linda Vahala, Jeffrey H. Harris, Glenn Bateman, and B. V. Waddell, *"Perturbed Magnetic Field Phase Slip for Tokamaks"*
- A20 George Vahala, Linda Vahala, Jeffrey H. Harris, Bruce Waddell, and Glenn Bateman, *"Perturbed Magnetic Field Phase Slip for Tokamaks"*

POSTER SESSION B  
(1:45-3:00 PM)

- B1 C. O. Beasley, Jr., D. J. Sigmar, W. I. van Rij, and H. K. Meier, *"Electrical Conductivity of Tokamak Plasmas Calculated by CPM Codes"*
- B2 S. H. Brecht, D. A. Hitchcock, W. Horton, and J. G. Lominadze, *"Ion Cyclotron Instability Driven by Neutral Beam Injection"*
- B3 C. Choi, G. Miley, and C. Bathke, *"A Unified Slowing-Down Theory for Fusion Plasmas"*
- B4 C. M. Celata and D. A. Boyd, *"Anisotropy Effects on the Cyclotron Radiation Emitted by Tokamak Plasmas"*
- B5 Tim Cutler and David Kershaw, *"The First Fully Implicit 2-D Fokker Planck Solver Using ICCG Algorithm"*
- B6 F. L. Hinton and J. C. Wiley, *"Self-Consistent Calculation of Oxygen Impurity Diffusion in Tokamak Plasmas"*
- B7 M. H. Hughes, *"A Numerical Model of ALCATOR"*
- B8 M. H. Hughes and D. E. Post, *"A Monte Carlo Algorithm for Calculating Neutral Gas Transport in Tokamaks"*
- B9 Thomas B. Kaiser, *"A Technique for Monte Carlo Simulation of Velocity Diffusion in Two Dimensions"*
- B10 T. B. Kaiser, *"Monte Carlo Simulation of a Quasilinear Transport Model for Mirror Plasmas"*

- B11 H. H. Klein, *"Recent Results of Tokamak Modeling"*
- B12 F. B. Marcus and S. E. Attenberger, *"Initial Phase of Tokamak Discharges"*
- B13 T. Matsuda, H. L. Berk, and J. Stewart, *"Quasilinear Model of a Mirror Plasma with Radial Transport"*
- B14 Allan T. Mense and S. E. Attenberger, *"Profile Effects on Plasma Transport in Tokamak Fusion Reactors Under the Influence of Trapped Particle Modes"*
- B15 A. A. Mirin, D. L. Jassby, M. G. McCoy, and J. Killeen, *"Fokker-Planck/Transport Simulations of Counterstreaming Ion Plasmas in the PDX Experiment"*
- B16 M. E. Rensink, Ronald H. Cohen, T. A. Cutler, and A. A. Mirin, *"Particle Confinement by Ambipolar Potentials in Mirror Machines"*
- B17 T. D. Rognlien, T. A. Brengle, and T. A. Cutler, *"Computer Model of Gas Feed Experiments in Mirror Machines"*
- B18 T. Uckan, E. C. Crume, S. P. Hirshman, and D. K. Lee, *"Numerical Evaluation of Multi-Impurity Pfirsch-Schluter Transport"*
- B19 F. Werkoff and C. Mercier, *"Energy Balance and Impurities in the Simulation Code MAKOKOT"*
- B20 J. C. Wiley and F. L. Hinton, *"A Comment on the Numerical Solution of Neoclassical Transport Equations"*

## POSTER SESSION C

(3:30-4:45 PM)

- C1 D. Biskamp and H. Welter, *"Linear and Nonlinear Properties of the M=2 Tearing Mode in Cylindrical Plasmas"*
- C2 Joon Y. Choe, Ronald C. Davidson, and William Grossmann, *"Stability Properties of an Anisotropic Guiding Center Plasma and Relation with the Suydam Function"*
- C3 R. Y. Dagazian and J. P. Freidberg, *"Diffuse Toroidal Helical Equilibria"*
- C4 R. Y. Dagazian, *"Nonlinear Effects on Free Boundary in Slab Geometry"*
- C5 R. A. Dory, D. B. Nelson, Y-K. M. Peng, D. Berger, L. Bernard, R. Gruber, and F. Troyon, *"Application of Ideal MHD Stability Code ERATO to High Beta Tokamak Equilibria"*

- C6 J. L. Fisher and J. E. McCune, *"Analytical Study of the Dynamical Development of Tokamak Equilibria in the Drift-Kinetic Approximation"*
- C7 H. Glad and E. Hameiri, *"Non-Resistive Changes in Field Topology"*
- C8 L. S. Hall, *"Magnetohydrodynamic Instability of Straight Axis Mirror Quadrupole Wells with Steep Radial Pressure Gradients"*
- C9 P. C. Liewer and C. S. Liu, *"Kinetic Theory of Ballooning Modes in Tokamaks"*
- C10 S. C. Jardin, *"Stabilization of Axisymmetric Instabilities in Noncircular Cross Section Tokamaks by Passive Feedback"*
- C11 R. A. Koch, P. H. Rutherford, and W. M. Tang, *"Collisional Effects for Current Driven Drift Waves"*
- C12 Y. C. Lee, J. F. Drake, and C. S. Liu, *"Semi-Collisional  $m = 1$  Internal Kink Modes"*
- C13 D. Montgomery and G. Vahala, *"Three Dimensional MHD Turbulence in Cylindrical Fusion Plasmas"*
- C14 Young-ping Pao, *"Nonlinear Behavior of Linearly Unstable Magnetohydrodynamic Modes"*
- C15 Young-ping Pao and Philip Rosenau, *"Nonlinear Behavior of Linearly Unstable MHD  $m = 2$  Mode"*
- C16 P. L. Pritchell, C. C. Wu, and J. M. Dawson, *"Investigation of Linear and Nonlinear MHD Internal Instabilities"*
- C17 W. M. Sharp, *"Generalized Guiding Center Equilibrium Equations"*
- C18 J. L. Shohet, J. N. Talmadge, and O. L. Betancourt, *"MHD Equilibrium of a Non-Circular Cross Section Stellarator"*
- C19 H. Weitzner, *"A Study of Mirror Machine Stability Problems"*
- C20 T. F. Young, *"The MHD Equilibria of a Separatrix Limited Doublet"*

POSTER SESSION D  
(3:30-4:45 PM)

- D1 R. E. Aamodt and M. C. Vella, *"Applications of the Kinetic Description of Ponderomotive Effects in a Plasma"*
- D2 John R. Cary and Allan N. Kaufman, *"Pseudomagnetic Field and a Relation Between Ponderomotive Force and Linear Susceptibility in Vlasov Plasma"*
- D3 B. I. Cohen, J. A. Byers, W. C. Condit, and J. D. Hanson, *"Hybrid Simulations of Quasineutral Phenomena in Magnetized Plasma"*
- D4 V. K. Decyk, J. M. Dawson, and G. J. Morales, *"Simulation of Lower-Hybrid Heating of a Finite Size Plasma"*
- D5 R. R. Dominguez and H. L. Berk, *"Variational Method for a Cylindrical Plasma"*
- D6 S. Peter Gray, *"Electromagnetic Ion Beam Instability and Energy Loss of Fast Alpha Particles"*
- D7 S. Hamasaki, N. A. Krall and J. B. McBride, *"The Effects of Turbulent Heat Flow on Implosion Heating Calculations"*
- D8 Robert W. Huff, *"Plasma Simulation on the CHI Computer"*
- D9 C. F. F. Karney and A. Bers, *"Stochastic Ion Heating by Lower Hybrid Waves"*
- D10 J. N. Leboeuf, T. Tajima and J. M. Dawson, *"Fluid Simulation Using Finite Size Particles"*
- D11 Kim Molvig, Miloslav S. Tekula, and Abraham Bers, *"Theory of the Runaway Electron Tail"*
- D12 M. Mond and G. Knorr, *"Elementary Derivation of the Kinetic Equation for the Two Dimensional Guiding Center Plasma"*
- D13 Reagan W. Moore, *"Shear Effects on Low-Frequency Geometric Resonance Heating"*
- D14 Edward Ott, Paul T. Donoli, and Jean-Marie Wersinger, *"Alfven Resonance Contribution to the Cavity Q in Low Frequency Magnetosonic Wave Heating"*
- D15 Bernard Rosen and Herbert Tesser, *"Automated Calculation of Parametric Couplings"*

- D16 A. Sen, C. F. F. Karney, and A. Bers, *"Two Dimensional Depletion of Lower Hybrid Pump by Quasi-Mode Excitations"*
- D17 A. Sen, C. F. F. Karney, G. L. Johnston, and A. Bers, *"Three-Dimensional Effects in the Nonlinear Evolution of Lower Hybrid Cones"*
- D18 A. G. Sgro, C. W. Nielson, Rogert D. Bergtson, and Stephen A. Eckstrand, *"Plasma Rotation Generated During a Theta Pinch Implosion"*
- D19 M. S. Tekula, K. Molvig, and A. Bers, *"Momentum Space Structure of the Steady State Runaway Distribution Function"*
- D20 Leak Turner, *"Oblique Magneto-Acoustic Heating of a Collisionless Pinch"*

THURSDAY, MAY 5

PLENARY SESSION: C. S. Liu and D. Sigmar, Chairmen

- PB1 } 8:40 AM T. Dupree, *"Strong Turbulence"*  
PB2 }
- PB3 Richard F. Post, *"On Field Reversal Via Neutral Injection"*
- PB4 A. Bers, *"Report on the Lower Hybrid Heating Theory Workshop"*
- PB5 Liu Chen, R. L. Berger, J. G. Lominadze, M. N. Rosenbluth, and P. H. Rutherford, *"Nonlinear Saturation of the Dissipative Trapped-Electron Instability"*
- PB6 C. S. Liu, Y. Mok, K. Papadopoulos, F. Engelmann, and M. Bornatici, *"Nonlinear Dynamics of Runaway Electrons in Tokamaks"*
- 10:20-10:40 AM COFFEE BREAK
- PB7 F. W. Perkins, *"Heating Tokamaks via the Ion-Cyclotron and Ion-Ion Hybrid Resonances"*
- PB8 Thomas E. Cayton, Richard A. Gerwin, and Leaf Turner, *"Magneto-Acoustic Heating of High Beta Pinches"*
- PB9 Dwight R. Nicholson and R. E. Aamodt, *"Theory of RF Bursting in Mirror Plasmas"*
- PB10 C. Z. Cheng and H. Okuda, *"Formation of Convective Cells and Anomalous Diffusion Due to Drift Turbulence"*

POSTER SESSION E  
(1:45-3:00 PM)

- E1 T. Amano and E. C. Crume, *"Numerical Calculation of Oxygen and Iron Impurity Diffusion in Tokamaks"*
- E2 G. Ara, B. Basu, and B. Coppi, *"Effects of Ion-Ion Collisions and Quenching of Sawtooth Oscillations in High Density Regimes"*

- E3      A. H. Boozer and A. Rechester, *"Effect of Field Errors on Divertor Scrape-Off Width"*
- E4      K. Borrass and D. Pfirsch, *"Diffusive Tokamak Equilibria, Generalized Bootstrap Currents and the Relation Between Particle Diffusion and Magnetic Field Diffusion"*
- E5      N. Byrne and M. Cotsaftis, *"Nonlinear Dynamics of Impurities in a Tokamak"*
- E6      M. Caplan and J. M. Dawson, *"Diffusion of Magnetic Field Lines in a Cylindrical Plasma Carrying Current"*
- E7      B. Coppi and C. Spight, *"Ion-Mixing Mode and Model for Density Rise in Confined Plasmas"*
- E8      M. Cotsaftis, N. A. Krall, and J. McBride, *"Nonlinear Transport in Tokamak Systems"*
- E9      R. D. Hazeltine and A. A. Ware, *"Neoclassical Theory of Equilibrium and Transport in a Z-Pinch Plasma"*
- E10     John T. Hogan, *"Injection, Compression and Transport Studies"*
- E11     D. L. Jassby, R. R. Parker, H. H. Towner, and R. J. Goldston, *"In the Matter of the Missing Neutrons from Alcator and TFR"*
- E12     P. L. Mascheroni, *"Transport During Turbulent Heating in a Tokamak"*
- E13     G. H. Miley, T. Petrie, L. Hively, P. R. Meka and M-Y. Wang, *"Fusion Product Transport and Slowing in Fusion Plasmas"*
- E14     D. E. Post, R. V. Jensen, and D. L. Jassby, *"Impurity Radiation Effects in Tokamaks"*
- E15     A. B. Rechester, *"Electron Heat Transport in a Tokamak with Destroyed Magnetic Surfaces"*
- E16     Mario Soler, *"Determination of Electron Heat Diffusivity from the Propagation of Sawtooth Heat Pulses in Tokamaks"*
- E17     J. L. Sperling, *"Quasilinear Theory and Enhanced Transport by Lower-Hybrid Waves"*
- E18     Loren Steinhauer, *"Classical and Anomalous Transport in Plasma Columns"*
- E19     R. N. Sudan and M. Keskinen, *"Theory of Strongly Turbulent Convection of Two-Dimensional Low-Pressure Plasma"*
- E20     Ronald E. Waltz, *"Transport Calculations for a Liner Compressed Plasma"*

POSTER SESSION F  
(1:45-3:00 PM)

- F1 Robert W. Bass, *"Topolotron's Unique, Maximal Immunity to Global Interchange Instabilities"*
- F2 F. W. Chambers and E. P. Lee, *"Unstable Modes of a Fully Pinched Relativistic Electron Beam in a Plasma"*
- F3 Bruno Coppi, *"Compact Experiments for  $\alpha$ -particle Heating"*
- F4 R. Y. Dagazian, *"Internal Ring End-Stopper for an Open Ended Device"*
- F5 G. C. Goldenbaum, Y. G. Chen, C. Chin-Fatt, Y. P. Chong, A. W. DeSilva, H. R. Griem, R. A. Hess, R. L. Merlino, and D. P. Murphy, *"Heating and Bootstrap Current Generation in the Thor High Beta Tokamak Experiment"*
- F6 E. Hameiri, *"The Stability of the Tormac Equilibrium"*
- F7 J. R. Hiskes, A. Karo, and M. Gardner, *"Negative Ion Yields from Hydrogen Atom Bombardment of Cesium-Coated Surfaces"*
- F8 Irvin R. Lindemuth and Thomas R. Jarboe, *"Initial Studies of Z-Pinch Plasma Under Liner Implosion Conditions"*
- F9 Robert C. Malone and Richard L. Morse, *"Material End-Plugging of Linear Theta Pinches"*
- F10 W. Manheimer and V. Granatstein, *"Application of Powerful Millimeter Wave Sources to the Tokamak Program"*
- F11 A. Mondelli and N. Rostoker, *"Confinement of Non-Neutral Plasma in a Torus with Applications to High Intensity Particle Accelerators"*
- F12 D. Nielsen, J. Green, and O. Buneman, *"Evolution of a Z-pinch as Revealed by a 3-D, e-m Particle Code"*
- F13 S. N. Rasband, *"A Diffuse Profile Topolotron Plasma Configuration"*
- F14 J. W. Shearer, *"Electron Energy and Space Charge Calculations in Reflex Diodes"*
- F15 J. L. Shohet, *"State of Stellarator Research"*

- F16 R. N. Sudan, M. J. Gerver, and G. O. Ludwig, *"High-Frequency Stability of a Field-Reversed Ion Ring in a Background Plasma"*
- F17 William B. Thompson and Siebe Jorna, *"Heavy Ion Induced Inertial Fusion: The Beam Propagation Problem"*
- F18 N. A. Uckan and C. L. Hedrick, *"Operating Point Considerations for the Elmo Bumpy Torus Reactor (EBTR)"*
- F19 M. Y. Wang and G. H. Miley, *"Orbit Calculation for Field Reversal Mirror"*
- F20 D. C. Watson and D. E. Baldwin, *"Alfven-Ion Cyclotron Instability in the Central Cell of TMX"*

POSTER SESSION G  
(3:30-4:45 PM)

- G1 P. Bakshi, W. Bellew, G. Ganguli, and P. Satyanarayana, *"Particle Orbits in Sheared Magnetic Fields and Drift Wave Instabilities"*
- G2 K. R. Chu, E. Ott, and W. Manheimer, *"The Dissipative Trapped-Electron Mode in Vertically Elongated Tokamaks"*
- G3 J. F. Clarke, *"An Improved Estimate of the Trapped Ion Mode Energy Loss from Tokamak Reactors"*
- G4 B. I. Cohen and W. M. Tang, *"2D Nonlinear Saturation of the Trapped-Ion Mode by Mode Coupling"*
- G5 J. Denavit and C. E. Rathmann, *"A Derivation of the Dissipative Trapped Electron Instability"*
- G6 P. N. Guzdar and P. K. Kaw, *"Effect of Random Potential Fluctuations on Drift Waves"*
- G7 W. Hobbs, W. Tang, W. Manheimer, and G. Rewoldt, *"Poloidal Structure of the Trapped-Electron Modes Including Curvature Drifts"*
- G8 W. Horton and Duk-In Choi, *"Nonlinear Effects in Drift Waves"*
- G9 J. Hsu, Liu Chen, and P. K. Kaw, *"Microscopic Magnetic Islands Due to Alfvén Instability in a Nonuniform Plasma"*
- G10 W. W. Lee and H. Okuda, *"A Finite- $\beta$  Simulation Model for Low-Frequency Instabilities"*

- G11 Stefano Migliuolo and Albert Simon, *"Nonlinear Saturation of the Trapped Electron Instability"*
- G12 William H. Miner and David W. Ross, *"Effects of Shear and Density Profile on the Spatial Structure of the Trapped Electron Mode"*
- G13 W. M. Nevins, *"A Physical Interpretation of Dissipative Drift Instabilities"*
- G14 A. B. Rechester, *"Adiabatic and Nonadiabatic Particle Motions, Resonance Overlapping and Stochastic Diffusion"*
- G15 C. Renaud, *"On the Contribution of the Dissipative Trapped Electron Instability to the Energy Balance in TFR"*
- G16 G. Rewoldt, W. M. Tang, and E. A. Frieman, *"Integral Formulation for the Two-Dimensional Spatial Structure of Drift and Trapped-Electron Modes"*
- G17 Gary R. Smith, *"The Motion of Ions in the Presence of a Trapped-Ion Mode of Finite Amplitude"*
- G18 K. T. Tsang and J. D. Callen, *"Anomalous Transport Due to Drift Waves in Tokamaks"*
- G19 A. K. Sen, *"Nonlinear Theory of Trapped Electron Turbulence"*
- G20 E. Valeo, *"Mode Coupling Saturation of Drift Waves"*

## POSTER SESSION H

(3:30-4:45 PM)

- H1 T. Antonsen and W. Manheimer, *"Electron Cyclotron Heating in Tokamaks"*
- H2 V. S. Chan and Charles Moeller, *"Slow Wave Launching at the Lower Hybrid Frequency Using an Electrostatic Antenna"*
- H3 Cheng Chu and J. L. Sperling, *"Cavity Eigenmode Heating of Tokamak Plasmas"*
- H4 E. H. da Jornada, J. D. Gaffey, Jr., and M. Z. Caponi, *"Anomalous Plasma Heating by Electrostatic Waves Driven Unstable by Neutral Beam Injection"*
- H5 R. W. Harvey and J. L. Sperling, *"Stochastic Heating of Electrons in Spatially Localized RF Fields"*

- H6 D. W. Hewett and C. W. Nielsen, *"Simulation of the LHD Instability Using a 2-D Hybrid Code"*
- H7 B. H. Hui and N. Winsor, *"Runaway Electrons in Collective Electric Fields"*
- H8 H. Ralph Lewis, *"Magnetoacoustic Heating in a Sharp-Boundary Pinch"*
- H9 A. T. Lin, *"Computer Simulations of Drift Alfven Wave Instabilities"*
- H10 Ronald L. Miller and George H. Miley, *"Monte Carlo Techniques for Neutral Beam Injection Studies"*
- H11 H. E. Mynick, *"Nonlocal Corrections to Poisson's Equation in Nonuniform Plasmas with Finite Gyroradius"*
- H12 H. Okuda, R. L. Berger, and C. Z. Cheng, *"Anomalous Heating of Magnetically Confined Plasmas Near the Lower Hybrid Frequency"*
- H13 R. S. Schneider and J. D. Gaffey, Jr., *"Collisional Heating by Alpha Particles and the Time Evolution of the Alpha Particle Distribution in Fusion Reactors"*
- H14 Thomas Speziale and Peter J. Catto, *"Magnetic Field Generation by Resonant Absorption in a Cold, Collisionless Plasma"*
- H15 D. G. Swanson, *"Cyclotron Harmonic Absorption Via the Green's Function for the Mode Conversion-Tunneling Equation"*
- H16 William B. Thompson and Alain Sciacaluga, *"Scattering of Cyclotron Radiation by Bernstein Modes"*
- H17 L. T. Wang, A. T. Lin, and J. M. Dawson, *"Computer Simulation of Synchrotron Radiation in a Magnetized Plasma"*
- H18 E. S. Weibel, A. Baños, Jr., and J. Van Dam, *"Langmuir Waves in a Spatially Periodic Potential"*
- H19 E. S. Weibel, *"The Fluid Theory of the Pondermotive Effect in a Magnetized Plasma"*
- H20 D. Winske, *"Particle Simulation Studies of the Lower Hybrid Drift Instability"*

5:00-5:40 PM R. PRICE, MFE THEORY PROGRAM

FRIDAY, MAY 6

PLENARY SESSION: R. Hazeltine and A. Kaufman, Chairmen

30-10

PC1 } 8:40 AM  
PC2 } 9:15

PC3 9:20

PC4 9:40

PC5 10:00

10:20

PC6

10:20-10:40 AM

PC7

PC8

PC9

PC10

D. Baker, "Toroidal Z-Pinch"

<sup>NAS.</sup>  
John M. Dawson, "Multipoles as Reactors"

B. Grant Logan, "The Tandem Mirror Concept"

N. T. Gladd, Y. Goren, C. S. Liu, and R. C. Davidson,  
"Influence of Magnetic Shear on the Lower-Hybrid-  
Drift and Mirror-Drift-Cone Instabilities with  
Application to Sheath Stability in Reversed-Field-  
Pinch (ZT) and Toroidal Bi-Cusp (TORMAC) Configura-  
tions"

Young-ping Pao, "Diffusion in Large-Aspect Toroidal  
Plasmas"

COFFEE BREAK

J. D. Callen, G. G. Kelley, K. T. Tsang, and  
B. V. Waddell, "Drift Wave Turbulence Effects on  
Magnetic Structure and Plasma Transport in Tokamaks"

H. L. Berk, T. D. Rognlien, and J. J. Stewart,  
"Quasilinear Simulation of Mirror Machines"

H. C. Chan and D. J. Sigmar, "Anomalous Transport  
of Alpha-Particles in Toroidal Fusion Plasma Due to  
Shear Alfvén Wave"

A. A. Ware, R. D. Hazeltine, and J. C. Wiley,  
"A Resonance Effect for Impurity Ions in Tokamak  
Plasmas"

## POSTER SESSION I (1:45-3:00 PM)

I1 D. B. Batchelor, "Recent Results in Microwave Heating  
Theory for EBT"

I2 D. B. Batchelor and C. L. Hedrick, "A Preliminary  
Look at Possible Trapped Particle Modes in EBT"

- I3 J. A. Byers and R. P. Freis, *"Computer Simulation of Field Reversed Mirror Configurations with the Superlayer Code"*
- I4 Ronald H. Cohen, M. E. Rensink, and A. A. Mirin, *"Tandem Mirror Confinement Studies"*
- I5 J. M. Finn and R. N. Sudan, *"Numerical Study of the Stability of Field-Reversed Ion Rings"*
- I6 A. Friedman, R. N. Sudan, and J. Denavit, *"A Linearized Three Dimensional Code for the Study of Strong Ion Ring Stability"*
- I7 W. Grossmann and M. Wakatani, *"End Loss Reduction in Linear Devices by Means of Oscillating Multiple Mirrors"*
- I8 E. F. Jaeger, *"Radial Transport with a Self-Consistent Electric Field in the Elmo Bumpy Torus"*
- I9 J. Kappraff, W. Grossmann, and P. Rosenau, *"Effect of Plasma Rotation and Hall Current on Linear Tearing Modes in Reverse Field Pinches"*
- I10 J. K. Lee and C. K. Birdsall, *"Ring-Plasma Electrostatic Instability; Linear Theory and Classification of Regimes; Simulation, Verification and Nonlinear Behavior"*
- I11 J. B. McBride, N. A. Krall, and A. L. Sulton, *"Thermal Stability of EBT Operating Points from a Neoclassical Point Model"*
- I12 M. A. Mostrom and A. H. Boozer, *"Rotational Damping in Tormac"*
- I13 C. G. Tull, J. Killeen, and M. E. Rensink, *"A Steady State Guiding Center Model for the Counterstreaming Ion Torus (CIT)"*
- I14 J. H. Whealton, *"Reduction in Residual Aberration Effects on Ion Optics in Neutral Beam Generators"*
- I15 F. Winterberg, *"Acceleration of a Magnetically Insulated Foil To Ultrahigh Velocities"*
- I16 James A. Rome and Theodore G. Northrop, *"Comments on Guiding Center Motion in High  $\beta$  Tokamaks"*
- I17 D. A. Spong, E. G. Harris, and C. L. Hedrick, *"Neoclassical Transport Coefficients for the ELMO Bumpy Torus Device"*
- I18 J. H. Wealton, *"Reduction in Residual Aberration Effects on Ion Optics in Neutral Beam Generators"*
- I19 James T. Woo, *"Ambipolar Potential of Tandem Mirrors"*

POSTER SESSION J  
(1:45-3:00 PM)

- J1 T. H. Dupree and D. J. Tetreault, *"A Renormalized Dielectric Function for Collisionless Drift Wave Turbulence"*
- J2 R. J. Goldston and H. H. Towner, *"Ripple Assisted Neutral Injection into Present and Future Tokamaks"*
- J3 Robert K. Golka and Robert W. Bass, *"Tesla's Ball Lightning Theory, A BGK-Wave Soliton-Pair Langmuir-Debye Sheath Surface-Tension Hypothesis, the Farnsworth Effect, and the "Hydrotron" Electrostatic-Inertial Self-Confined Plasmoid Concept"*
- J4 Akira Hasegawa and Kunioki Mima, *"Stationary Spectrum of Strong Turbulence in Magnetized Nonuniform Plasma"*
- J5 D. A. Hitchcock, S. H. Brecht, and W. Horton, Jr., *"Convective Loss Cone Instability Driven by Energetic Ion Beams"*
- J6 S. M. Mahajan, P. N. Guzdar, P. K. Kaw, and C. Oberman, *"Drift Convective Cells in a Sheared Magnetic Field"*
- J7 Oscar P. Manley, *"The Vlasov-Poisson Equations - Another View"*
- J8 K. Mima and Y. C. Lee, *"Two-Dimensional Localization of Nonlinear Drift Wave and Formation of Convective Cells"*
- J9 E. Ott and W. Manheimer, *"Tokamak Heating with Intense Pulsed Ion Beams"*
- J10 L. D. Pearlstein and N. Maron, *"Stability Boundaries for Loss-Cone Modes"*
- J11 N. R. Pereira, *"Soliton in the Damped Nonlinear Schroedinger Equation"*
- J12 G. Schmidt and Y. Satya, *"Drift Wave Coupling via Stimulated Compton Scattering"*
- J13 H. R. Strauss, *Nonlinear Motion of High Beta Tokamaks"*
- J14 T. Tajima, K. Mima, and J. M. Dawson, *"The Alfven-Ion Cyclotron Instability"*
- J15 Yvain M. Treve, *"A Mathematical Criterion for Stochasticity with Application to Ion Heating"*
- J16 S. Yoshikawa, *"What is the Critical  $\beta$  for Ballooning Mode"*

PLENARY SESSION PA

PA1, PA2

T. K. CHU - LINEAR MAGNETIC FUSION  
(NO ABSTRACT)

## TEARING MODES IN TOKAMAKS\*

B. V. Waddell, J. D. Callen, B. Carreras,  
H. R. Hicks, J. Holmes, and G. L. Jahns  
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

Two aspects of the study of tearing modes in Tokamaks will be discussed: (1) internal disruptions and (2) interaction of modes of different helicity. The motivation for studying the second problem is to determine if such interactions are a mechanism for the major (external) disruption.

With respect to internal disruptions (or sawtooth oscillations), a self-consistent model which explains ORMAK data has been devised. The fundamental hypothesis is that the slow rise of a given sawtooth is due to resistive heating and the disruption is due to the nonlinear growth of the  $m = 1$  tearing mode.<sup>1</sup> The model, which includes the effect of diamagnetic drifts on the growth rate, is successful in explaining, among other things, the growth rate of the  $m = 1$  precursor oscillations, the rate at which the growth rate changes, and the repetition time of the sawteeth.

In order to study the interaction of tearing modes of different helicity, a reduced set of three-dimensional resistive MHD equations has been devised. Furthermore, a numerical algorithm for solving these equations has been devised and is presently being tested. In addition, semi-analytic studies of the linear and non-linear coupling of tearing modes are being conducted.

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\*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>1</sup>B. V. Waddell, M. N. Rosenbluth, D. A. Monticello, and R. B. White, Nucl. Fusion 16, 528 (1976).

## Drift-Tearing Instabilities due to Trapped Electron\*

Liu Chen, P. H. Rutherford and W. M. Tang

Plasma Physics Laboratory  
Princeton University  
Princeton, New Jersey 08540

It is shown that the collisional detrapping of magnetically trapped electrons in tokamaks can excite drift-tearing instabilities with high azimuthal wavenumbers, typically as high as  $m \sim 100$ . The modes are driven unstable by the expansion free energy associated with density and temperature gradients. For this reason, instabilities can occur even though the usual tearing modes driven by the magnetic free energy are stable, i.e., even if  $\Delta' < 0$ . For normal temperature gradients ( $d \ln T_e / d \ln n > 0$ ), the trapped-electron tearing modes are found to be unstable in the more collisional regime ( $\nu_{\text{eff}} > \omega_{*e}$ ), but stable in the less collisional regime ( $\nu_{\text{eff}} < \omega_{*e}$ ). The modes have oscillation frequencies near  $\omega_{*e}$ . Since the modes will cause the formation of magnetic islands, they may lead to significant anomalous cross-field transport, by mechanisms different from those involving only electrostatic trapped-electron modes.

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\*Work supported by U. S. ERDA Contract No. E(11-1)-3073.

# KINETIC THEORY OF TEARING INSTABILITIES

J. F. Drake and Y. C. Lee

Center for Plasma Physics and Fusion Engineering, UCLA

Los Angeles, California 90024

We present a unified theory of tearing modes valid in plasma with arbitrary collision frequency  $\nu$  and arbitrary diamagnetic frequency  $\omega^*$ . The structure of the current layer around  $k \cdot B = 0$  and the growth rate of the instability separate into three regimes, depending on the collisionality of the plasma. In the collisionless and semi-collisional regimes, the current layer width  $\Delta$  is governed by electron thermal motion along  $B_0$ , while in the collisional limit  $\Delta$  is determined by the condition  $E_{||} = 0$ . Previous theories are valid only in the collisional limit. The temperature gradient destabilizes the tearing mode in the collisional and semi-collisional regimes and numerical calculations indicate that drift-tearing modes ( $\omega \approx \omega_e^*$ ) of up to  $m \approx 60$  may be unstable in PLT. The role of these high order modes in energy transport will be discussed. Low  $m$  modes with  $\omega \approx \omega_i^*$  are also found to be unstable.

For  $\omega^* \approx 0$ , nonlinear effects modify the growth of the tearing mode when the island width  $w \approx \Delta_L$ ,  $\Delta_L$  being the linear tearing width. The collisionless tearing mode saturates when  $w \approx \Delta_L$ . In the semi-collisional regime thermal motion of electrons around the new magnetic surfaces causes  $\Delta = w$  in the nonlinear stage, leading to a secular growth of  $w$  as in the collisional calculation of Rutherford. Saturation of the tearing mode in slab geometry is also discussed.

This work supported by ERDA and NSF.

## STABILIZATION OF DISRUPTIVE INSTABILITY

## BY RESONANT HELICAL WINDINGS

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 Euratom Association, D-8046 Garching,  
 Federal Republic of Germany

Application of external resonant  $\ell = 2$  windings has been shown experimentally to lead for a certain regime of external winding currents  $I_{hel}$  to a quenching of the  $m = 2$  oscillations and a retardation in the onset of the disruptive instability /1/,/2/. We suggest that this effect is due to the local enhancement of radial heat conduction due to the magnetic islands produced by the resonant helical windings. The increased heat conduction leads to a flattening of the temperature and current profile around the  $q = 2$  surface with a stabilizing effect on the  $m = 2, n = 1$  mode /3/.

Quantitative examinations were carried out by first fitting a heat conduction coefficient profile to the measured unperturbed temperature distributions. This coefficient was then enhanced over a region corresponding to the size of the magnetic islands and used to compute a modified temperature profile. The resulting current distribution, obtained by assuming  $j_{\varphi} \sim T^{3/2}$ , was then tested for stability against the  $m = 2, n = 1$  tearing mode. Results show that the island sizes produced in Pulsator by helical currents in the stabilizing range of  $I_{hel}$  should indeed suffice for suppressing this mode.

For values of  $I_{hel}$  in excess of a critical value, the region of increased heat conduction extends out to the limiter, leading not any more to a local plateau, but to a shrinking of the discharge and an induced disruption.

- /1/ F. Karger et al., 5th Conf.on Plasma Phys.and Contr.Nucl.  
Fusion Research, Tokyo, 1974, IAEA-CN-33/PD-2
- /2/ F. Karger et al., 6th Conf.on PlasmaPhys. and Contr.Nucl.  
Fusion Research, Berchtesgaden, 1976, IAEA,CN-35/A-7
- /3/ A.H. Glasser, H.P.Furth and P.H.Rutherford, Phys.Rev.Letters  
38 (1977) 234

THEORETICAL AND EXPERIMENTAL STUDIES OF MHD STABILITY IN A FINITE-BETA  
RECTANGULAR CROSS SECTION TOROIDAL PLASMA DEVICE

P. C. Liewer, A. W. Allen, F. L. Cochran, and G. C. Goldenbaum  
University of Maryland, College Park, Maryland 20742

The dependence of the growth rate of the  $n=1$  kink instability on safety factor  $q$  and plasma beta has been studied theoretically for a finite beta [ $\langle\beta_T\rangle=3-6\%$ ], rectangular cross section toroidal plasma. Results are compared to experimentally inferred growth rates of kink instabilities observed in the University of Maryland experiment, TERP.<sup>1</sup> Theory and experiment are in good agreement on the critical toroidal beta for stability as well as on the estimated values of the growth rate when the essential features of the experimental configuration (noncircular, aspect ratio  $R_0/a=2.0$  paramagnetic equilibria) are included in the model. The growth rates for the  $n=1$  (toroidal mode number) kink mode are calculated numerically using Bateman's linearized ideal MHD code.<sup>3</sup>

The experiment<sup>2</sup> has a rectangular cross section with elongation  $b/a=2.0$ , major radius  $R_0=15$  cm, plasma density  $n=2\times 10^{14}\text{ cm}^{-3}$ , and temperatures on axis  $T_e\approx T_i\approx 5$  eV. As the toroidal field is varied from 300 to 800 G, the density, temperatures and poloidal beta are observed to remain the same with  $\beta_{pol}\approx 0.7$ . As the toroidal field is lowered from  $B_T\approx 800$  G ( $\beta_{T,axis}\approx 10\%$ ) to  $B_T<500$  G ( $\beta_{T,axis}>20\%$ ) the plasma is observed to go from MHD stable to MHD unstable to an  $m=1$ ,  $n=1$  kink mode.

Similarly, in the theoretical studies, the growth rate of the  $n=1$  mode was studied as a function of  $B_T$  (hence  $\beta_T$  and  $q$ ) for a fixed  $\beta_{pol}=0.67$ . Both theory and experiment show a stable plasma ( $\gamma\tau_A<.01$ , where  $\tau_A=a/V_A$  where  $V_A$  is the toroidal Alfvén speed) for  $\beta_{T,axis}<15\%$  ( $\langle\beta\rangle_T<4\%$ ). For  $\beta_{T,axis}>15\%$ , both theory and experiment predict an  $n=1$ ,  $m=1$  kink instability. The agreement on estimated growth rates is good. For example, for  $\beta_{T,axis}\approx 20\%$  ( $\langle\beta_T\rangle\approx 6\%$ ) theory predicts  $\gamma\tau_A\approx 0.06$ . The experimentally inferred growth rate is  $\gamma\tau_A\approx 0.08-0.11$ .

1. A. W. Allen, F. L. Cochran, G. C. Goldenbaum, and P. C. Liewer (submitted for publication).
2. A. W. Allen and G. C. Goldenbaum, University of Maryland Preprint #607P029.
3. G. Bateman and Y. K. M. Peng, PRL 15, 829 (1977).

## Tokamak Stability Studies Using the PEST Code\*

M. S. Chance, J. M. Greene, R. C. Grimm, Y. Y. Hseih,  
 J. L. Johnson,<sup>†</sup> J. Manickam, M. Okabayashi,  
 A. M. M. Todd and D. E. Voss

Plasma Physics Laboratory  
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 Princeton, New Jersey 08540

The Princeton Equilibrium, Stability, and Transport Code PEST<sup>1,2</sup> is being used to investigate the stability properties of realistic configurations with respect to axisymmetric, kink, and ballooning modes. Recently, our major emphasis has been on the importance of convergence studies to ensure that the worst instabilities can be represented properly. In the course of this work a comparison<sup>3</sup> of results for a simple analytic model using different codes has been completed, studies<sup>4</sup> of ballooning mode limitations on beta in tokamaks with nearly circular cross sections have been refined, and evaluation<sup>5</sup> of instabilities in D-shaped devices has been extended. Continuation of the study of ballooning mode limitation on beta has shown an  $a/R$  dependence of  $\beta_C^*$  for both circular and D-shaped tokamaks. Variations of  $\beta_C^*$  with shape,  $q$  on the axis,  $q$  at the limiter, and the pressure profile are being investigated. Shaping can increase  $\beta_C^*$  by at least a factor of 2 to 3;  $\beta_C^*$  decreases if  $q_a$  is increased provided  $q_a$  is somewhat greater than 1.

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\* Work supported by U. S. ERDA Contract No. E(11-1)-3073.

<sup>†</sup> On loan from Westinghouse Research and Development Center.

1. R. C. Grimm, J. M. Greene, and J. L. Johnson, in Methods in Computational Physics (Academic Press, N.Y. 1976) Vol.16,253.
2. J. Manickam et al., Bull. Am. Phys. Soc. II 21 , 1079(1976).
3. W. Kerner et al., ibid.
4. A. M. M. Todd et al., ibid.
5. Y. Hseih et al., ibid.

Localized Ballooning Modes in Noncircular Cross-Sectioned Tokamaks.\*

D. Dobrott and D. Lulue, General Atomic Company--Magnetohydrodynamic stability is examined for general, high beta axially-symmetric equilibria by means of the energy principle for high toroidal mode number. In such a limit, stability analysis between surfaces is decoupled and the equilibrium poloidal flux appears only as a parameter. In the case of small shear, stability analysis is reduced to solving an Euler equation for the normal displacement with a poloidal angle-like parameter as the independent variable. Thus, all poloidal mode numbers have been included in the analysis. The results are complimentary to the Princeton PEST code. The orthogonal coordinate system used in the analysis is obtained numerically and may be adjusted easily for a given equilibrium. Stability for several equilibria are examined.

\*Work supported by the U. S. Energy Research and Development Administration, Contract EY-76-C-03-0167, Project Agreement No. 38.

# Ballooning Modes in Axisymmetric Toroidal Plasmas\*

A. H. Glasser, J. M. Greene, M. S. Chance  
and E. A. Frieman

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We study the linearized equations of ideal and resistive MHD in the limit of large mode numbers. An equilibrium magnetic field  $\vec{B}$  with finite shear is represented in terms of Hamada coordinates  $v, \theta, \zeta$ . Perturbed fields are represented in terms of potentials  $\vec{A}$  and  $\phi$ . We introduce the ordering

$$\frac{\partial}{\partial v} \sim \frac{\partial}{\partial \theta} \sim \frac{\partial}{\partial \zeta} \sim \epsilon^{-1}, \quad \frac{\partial}{\partial t} \sim \vec{B} \cdot \vec{\nabla} \sim 1, \quad \eta \sim \epsilon^2, \quad \epsilon \ll 1,$$

with  $t$  the time and  $\eta$  the resistivity. We solve the equation of motion and Ohm's law for the perturbed fluid velocity  $\vec{v}$  and current density  $\vec{j}$ , and use these solutions in Ampere's law, the quasineutrality equation, and the adiabatic law for the perturbed pressure  $p$ , to obtain a closed set of partial differential equations for  $\vec{A} \cdot \vec{B}$ ,  $\phi$ , and  $p$ . In the limit of zero resistivity, these reduce to two equations for  $\phi$  and  $p$ . In this limit of zero frequency, they reduce to one equation for  $\phi$ . The equation can also be obtained from the ideal MHD energy principle, which shows that the important terms involve a competition between destabilizing local curvature and stabilizing tension. In the limit of small mode numbers but large radial derivatives, the full set reduces to the equations for resistive instabilities, which describe tearing modes and ideal and resistive interchanges. Specializing to the single equation for  $\phi$ , we discuss a problem of single-valuedness which arises when we look for radially extended high- $m$  modes in sheared fields. We then describe a numerical calculation which determines marginal stability from the resulting 1-D equation, and compare the results with those obtained from the 2-D PEST code. Finally, we discuss a technique for obtaining marginal stability criteria in terms of quadratures.

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\*Work supported by U. S. ERDA Contract No. E(11-1)-3073.

## ADVANCES IN SCYLLAC EQUILIBRIUM THEORY\*

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The existence and properties of diffuse equilibria in a high- $\beta$  stellarator geometry such as Scyllac have continued to be investigated by analytic and numerical methods. Numerical results have been extended to show the existence of equilibria over a range of plasma  $\beta$ . These results have been compared to the scaling given by an analysis of toroidal forces and with experiments.

The numerical approach to MHD equilibrium has been studied for both  $\ell=1,0$  and  $\ell=1,2$  systems for  $\beta$  from .5 to 1. The model describes a fully three-dimensional toroidal MHD system in which a diffuse plasma region is separated from a vacuum field region by a free boundary.<sup>1</sup> A toroidal theta pinch is taken as initial conditions. Then helical fields are gradually added. For computing equilibria the toroidal motion of the column is constrained so that the center of mass remains on the axis of the system. The force of constraint indicates the toroidal force imbalance and decays to zero as equilibrium is approached. Dissipation can also be added in the form of artificial friction.

Numerical results for sharp boundary equilibria are in agreement with analytic theory. For diffuse profile plasmas, the toroidal force balance is compared with an analytic model<sup>2</sup> which combines the conditions for local force balance to the applied helical fields with the condition for average toroidal force balance. From this model it is possible to estimate in a non-self-consistent way the fields required to produce equilibrium. Good agreement of the numerical results with the analytic force estimate is obtained. Further, when toroidal force balance is produced, an equilibrium in the sense of local force balance is obtained.

The numerical results show that the scaling with aspect ratio predicted by expansions for small helical distortions is valid even for rather large plasma deformations. However, the scaling with  $\beta$  is found to be quite different for diffuse profiles compared with sharp boundary, in agreement with the analytic force estimate.<sup>2</sup> This difference is especially important for  $\ell=1,0$  systems at high  $\beta$ . These theoretical results have been compared with several experiments. For Scylla IV-3 and ISAR T1-B good agreement has been obtained. Most recently, the theoretical predictions have been tested in the  $\ell=1,0$ , Scyllac experiment by applying additional helical fields to the design fields which were based on sharp boundary theory. The value of the trimming field required to balance toroidal forces represents a 57% correction to sharp boundary predictions and agrees with the above diffuse theory results.

<sup>1</sup>D. C. Barnes and J. U. Brackbill, to appear in Nucl. Sci. & Engr. (July 1977).

<sup>2</sup>D. C. Barnes, to appear in Physics of Fluids

\*Work performed under the auspices of U.S.ERDA.

POSTER SESSION A

## Equilibrium Models for Field Reversal for 2XIIB Parameters

by

D. V. Anderson, M. E. Rensink and W. C. Turner

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Livermore, California 94550

### ABSTRACT

Diagnostics for directly measuring magnetic field magnitude and direction inside the 2XIIB plasma are under development at LLL. In the interim period we have developed indirect procedures for estimating the fields inside the 2XIIB plasma. Three 2D axisymmetric models 1.) Vlasov rigid rotor, 2.) Rotating tensor pressure, and 3.) Given current, will be presented. All of these models allow finite gyro orbits which is an essential feature of 2XIIB. In contrast our 3D guiding center equilibrium codes cannot represent finite gyro radii so they could not be used here. In these models the ions carry all the current. Some of the parameters for these models are derived from the available 2XIIB data; in particular line densities, energy spectra, and diamagnetic loop signals are used to estimate the parameter set to use in each model. Results of high beta equilibria will be presented for typical 2XIIB parameters. The parameters required for field reversal in 2XIIB will be discussed.

\*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

# Classical Diffusion at an X Point<sup>\*</sup>

S. P. Auerbach and A. Boozer

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Princeton, New Jersey 08540

We have studied classical diffusion of a plasma across an X point formed between two infinitely long parallel straight wires, separated by a distance  $2a$ , carrying current in the same direction, with a superposed uniform magnetic field  $B_z$  parallel to the wires. This model is exactly soluble in the low beta limit. The total particle flux across a magnetic surface is  $\Gamma = D(\psi) dP/d\psi$ , where  $\psi$  is the poloidal flux. We find that the diffusion coefficient  $D(\psi)$  is finite everywhere and varies slowly as the separatrix is approached. Consequently,  $dP/d\psi$  is finite at the separatrix. Approaching the separatrix from the outside,  $D(\psi)$  approaches  $8\pi^2 nc^2 a B_p/B_z^2$ , where  $B_p$  is the value the poloidal field would have a distance  $a$  from one wire if the other wire were absent. Just inside the separatrix,  $D(\psi)$  is half as large, because the magnetic surface splits into two surfaces at the separatrix. The cross-field diffusive flow, which is fairly uniform over the magnetic surfaces far from the separatrix, becomes concentrated close to the X point on surfaces near the separatrix.

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<sup>\*</sup> Work supported by EPRI Contract No. RP 423 - 1 and U. S. ERDA Contract No. E(11-1)-3073.

LARGE SCALE MHD STABILITY  
OF FCT TOKAMAKS\*

Glenn Bateman and Y-K. M. Peng  
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ABSTRACT

Sequences of equilibria with high average beta have been computed<sup>1</sup> with prescribed high-shear q-profiles, practical values of the aspect ratio (3-4), and controlled shape. We have been working with Flux Conserving Tokamak sequences of equilibria in which the q profile is held fixed. It has been found<sup>2,3</sup> that the added pressure drives a ballooning instability which cannot be avoided above a critical beta regardless of the q-value. Unlike the low beta  $m = 1$  instability which becomes localized near the magnetic axis as the marginal stability condition is approached, this ballooning instability persists near the edge of the plasma. It is observed to be sensitive to the boundary conditions used for the edge of the plasma and the state of the material between the plasma and the wall as well as the shape of the plasma and the profiles near the edge. This sensitivity to edge conditions and plasma shape is investigated with the Oak Ridge linear initial value instability code.

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\*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>1</sup>R. A. Dory and Y-K. M. Peng, Nucl. Fusion 17, 21-32, (1977)

<sup>2</sup>J.P. Freidberg and F. A. Haas, Phys. Fluids 17, 440, (1970)

<sup>3</sup>G. Bateman and Y-K. M. Peng, to appear in Phys. Rev. Lett.; A.M. M. Todd, et al, MATT - 1320

ANALYTIC TREATMENT OF THE INTERACTION  
OF TEARING MODES IN TOKAMAKS\*

B. Carreras and B. V. Waddell  
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

Employing the reduced set of three-dimensional resistive MHD equations<sup>1</sup>, three problems regarding the interaction of tearing modes are being studied semi-analytically. 1.) The nonlinear growth of modes with helicity  $m/n = 1$ , where  $m$  and  $n$  are respectively the poloidal and toroidal mode numbers. 2.) The nonlinear interaction of modes with different helicity in cylindrical geometry. 3.) The linear coupling of tearing modes of different helicity through the equilibrium field in toroidal geometry. With respect to the second problem, for example, it has been found that helicity  $m/n = 1$  coupling with helicity  $m/n = 3/2$  can drive helicity  $m/n = 2$ . Other preliminary results for the two remaining problems will also be presented.

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\*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>1</sup>H. R. Hicks, B. V. Waddell, B. Carreras, and J. Holmes, abstract submitted to this conference.

Magnetic Tearing Due to Finite- $\beta$  Modified  
Drift Waves<sup>\*</sup>

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We present a new simplified approach to the analysis of the stability and eigenmode structures of finite- $\beta$  modified drift waves in a sheared magnetic field. This approach is then applied to the collisionless universal drift mode. For the lowest ( $n=0$ ) radial eigenmode, we recover the previous result<sup>1</sup> that finite- $\beta$  effects stabilize the instability. However, for the  $n=1$  eigenmode, which has odd electric potential perturbations, we find the mode is further destabilized by the finite- $\beta$  effects. More interestingly, the corresponding eigenmode structures exhibit non-zero tearing-type magnetic perturbations ( $B_r \neq 0$ ) at the mode rational surface. Thus, instabilities of the  $n=1$  and other odd-number eigenmodes can lead to field line reconnection, and formation of magnetic islands with high azimuthal mode numbers, near the mode rational surfaces.

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<sup>\*</sup>Work supported by U. S. ERDA Contract No. E(11-1)-3073.

1. P. J. Catto, A. M. El Nadi, C. S. Liu and M. N. Rosenbluth, Nucl. Fusion 14, 405 (1974).

3-D NONLINEAR EVOLUTION OF THE INTERNAL KINK MODE  
IN A RESISTIVE PLASMA\*

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We present preliminary results from a new three dimensional, time dependant, implicit MHD code (IMP). IMP uses the Douglas-Gunn algorithm<sup>(1)</sup> to solve the complete set of MHD equations in full conservation form on a nonuniform, orthogonal curvilinear Eulerian mesh. The initial cases compare the effects of finite resistivity on the internal kink ( $m=1$ ) mode in a rectangular cross section pipe. The ideal MHD runs show good agreement with the explicit linear and nonlinear codes of Wooten, Hicks and Bateman.<sup>(2,3,4)</sup> The second problem run will be the internal kink ( $m = 1, n = 1$ ) in a rectangular cross section Tokamak of aspect ratio 4.

- 1) Douglas, J., and Gunn, J., A General Formulation of Alternating Direction Methods, Numer. Math. 6 (1964)
- 2) Bateman, G., Schneider, W., Grossman, W.; Nucl. Fusion 14 (1974)
- 3) Hicks, H.R., Wooten, J.W., Bateman, G.; Linear Instability Code, Bull. Am. Phys. Soc. 19 (1974)
- 4) Bateman, G., Hicks, H.R., Wooten, J.W.; 3-D Nonlinear Evolution of MHD Instabilities, (1974) submitted to Nuclear Fusion.

\*Work performed under the auspices of the U.S. Energy Research and Development Administration Contract No. W-7405-Eng-48.

Classical Diffusion and Energy Transport:  
Equations and Numerical Algorithms<sup>†</sup>

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In a large aspect, low  $\beta$  limit, Grad and Hogan showed that plasma diffusion and current penetration (skin effect) decouple and evolve on two distinct time scales. More generally, in other parameter ranges, the equations are both analytically and numerically nonstandard. A survey is given of the optimum forms for the equations - dependent and independent variables; types of algorithms, meshes, etc. - in most of the relevant parameter ranges. Free boundaries, limiters, and divertors are included. Doublet and tearing types of isolation require separate handling. As one example, a simulation with realistic parameters of a resistive transition from a belt pinch to a fully formed Doublet requires under one minute of 6600 time.

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<sup>†</sup> Work supported by U.S. ERDA Contract No. EY-76-C-02-3077.

LOCAL, IDEAL AND RESISTIVE STABILITY IN  
TOKAMAKS WITH NONCIRCULAR CROSS SECTION\*

B.J. Green, H.P. Zehrfeld

Recently, the conditions for stability of an arbitrarily shaped, finite pressure toroidal plasma against localized ideal and resistive modes were presented. The characteristic time scale for local instability with respect to ideal modes is small compared with that for resistive modes so that an investigation of ideal local stability is a prerequisite for an assessment of nonideal local stability. Here we consider the stability of a particular class of non-circular cross-section tokamak equilibria with respect to such modes. The equilibria are described analytically so that no ordering or expansion procedure is necessary, and both ideal and resistive stability is investigated over the whole plasma region. The effects of plasma shape (vertical elongation and triangularity), as well as aspect-ratio are investigated and particular reference is made to next generation tokamak designs (e.g. JET).

\*This work was performed under the EURATOM JET Design Phase Agreement 30-74-IFUA-C. B.J. Green is in the JET Design Team, Culham Laboratory, Abingdon, U.K, and at the Institut für Plasma-physik (IPP), Garching, FRG and H.P. Zehrfeld is at IPP Garching.

Stability Results on Racetrack Equilibria.\* F. Joanne Helton and Reagan Moore, General Atomic Company--MHD stability results obtained from the PEST code are presented for racetrack equilibria. Toroidal mode numbers up to  $n = 4$  and dependence upon  $q$  profile are considered. Although an internal separatrix cannot be modeled with the PEST code, effort is made to model Doublet IIA discharges. Elongations of up to 3:1 are considered. Plasmas with noncircular cross section exhibit higher thresholds for ballooning instabilities than plasmas with circular cross section.

\*Work supported by the U. S. Energy Research and Development Administration, Contract EY-76-C-03-0167, Project Agreement No. 38.

NONLINEAR EVOLUTION OF THE REDUCED SET OF  
THREE-DIMENSIONAL RESISTIVE MHD EQUATIONS FOR TOKAMAKS\*H. R. Hicks, B. V. Waddell, B. Carreras, and J. Holmes  
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A reduced set of low  $\beta$ , three-dimensional, resistive MHD equations has been derived including the effects of resistivity and toroidicity. These equations are the generalization in toroidal geometry of the equations employed in Ref. 1. We denote the toroidal and poloidal magnetic fields by  $B_\zeta$  and  $B_p$  respectively. The three basic assumptions are (1)  $\partial B_\zeta / \partial t = 0$ , (2)  $B_p / B_\zeta \sim \epsilon$ , and (3)  $\beta \sim \epsilon^2$ . The first assumption eliminates fast time scale magnetosonic waves, which greatly increases the efficiency of any numerical scheme based on a flux function, thereby insuring that  $\nabla \cdot \underline{B} = 0$  numerically. The assumptions imply that the toroidal velocity can be neglected and that the fluid is incompressible; this permits the introduction of a stream function for the poloidal velocity. Equations correct to first order in  $\epsilon$  are obtained for the time evolution of the toroidal flux and the vorticity. A numerical algorithm has been designed for studying the nonlinear evolution of these equations. Preliminary results on the interaction of tearing modes of different helicity will be reported.

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\*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

1. B. V. Waddell, M. N. Rosenbluth, D. A. Monticello, and R. B. White, Nucl. Fusion 16, 528 (1976).

## MHD Stability of High Beta Tokamak Equilibria with Force Free Currents

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Toroidal axisymmetric equilibria with plasmas surrounded by force free currents are numerically generated with a code developed at Columbia by M. F. Reusch. The stability calculations for high beta tokamaks of rectangular cross section are carried out by a time-dependent method similar to that of Wesson and Sykes<sup>1</sup>. The stabilizing effects of the force free currents on the internal MHD modes are studied. The growth rates of the fastest growing modes are plotted against the plasma beta for different strengths of the force free fields. Increasing the force free current outside a diamagnetic plasma will elongate slightly the cross section of the plasma along the major axis and flatten the q-profile. Subsequently, no internal kink modes ( $n = m = 1$ ) appear at a plasma beta less than 10%, where  $n$  and  $m$  are respectively the toroidal and poloidal mode numbers. Also, the internal modes with  $n = 1$  can be stabilized by the force free currents.

1. Wesson, J. A., Sykes, A., IAEA-CN-33/A12-3, Tokyo (1974).

This work was supported by U.S. ERDA under contract EY-76-S02-2456

PEST Documentation<sup>\*</sup>J. Manickam and B. McCann<sup>†</sup>

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A user oriented description of the PEST code is given. PEST (Princeton Equilibrium, Stability, and Transport) is a computer program package for studying ideal MHD instabilities in toroidal plasmas and is being made available to other researchers by PPL.

The presentation includes a description of the equations solved and the numerical techniques used with particular attention given to timing and accuracy scaling as this affects parameter searches and convergence studies. Attention is given to the peculiarities of the selective eigenvalue solver (VEKIT), and a comprehensive list of input parameters and their restrictions is shown.

Documentation and other user aids will be made available, and present users are invited to make suggestions on both documentation and aids.

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<sup>\*</sup>Work supported by U. S. ERDA Contract No. E(49-20)-1006.

<sup>†</sup>On loan from Science Applications, Inc., 8400 Westpark Dr., McLean, VA 22101.

# STATIONARY AXISYMMETRIC STATES WITH DIFFUSION AND HEATING OF TOKOMAK CONFIGURATIONS

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The aim of this work is the study of the axial-symmetry stationary states of a Tokomak plasma, taking into account anomalous transport coefficients and heating (Joule heating or other heatings).

Starting from the equilibrium MHD equations and assuming that the value of the heat transport coefficient  $K$  is  $K_R = n\eta$  ( $n$  density,  $\eta$  resistivity), one can then find to a good approximation :

$$E_0 = \frac{\varepsilon_0}{X} = \eta j_e = \eta_0 j_e / \tau^{3/2}$$

$X$  is the distance from the revolution axis. We call  $K_R$  the reference transport coefficient. Its value is near the value of  $K$  which has been empirically computed. If, moreover, one puts  $n = \xi T^\omega$ , (a formula which is well verified experimentally), one obtains the following equations : ( $\omega = 1/2$ )

$$\mathcal{L}.F = A(F) + (X_0^2 - X^2) \left\{ \frac{\eta_0}{E_0} \frac{dA}{dF} \right.$$

$$\left. \oint_C (X_0^2 - X^2) \frac{dl}{X |\text{grad} F|} = 0 \right.$$

$$\mathcal{L}. \equiv X^2 \text{div} X^{-2} \text{grad}.$$

where  $C$  is the cross-section meridian of the magnetic surface  $F$ . One uses the energy transport equation for electrons

$$\text{div} Q_e = E_e j_e + s$$

where  $s$  represents the supplementary heat sources. One chooses as transport coefficient the functional form

$$K \sim T^\alpha q^\beta n^\gamma \sim T^{(\alpha + \omega\gamma)} q^\beta \sim T^\alpha q^\beta$$

where  $q$  is the safety factor.

One can then find  $A$  and obtain the functional equation to be solved

In the surface case of ohmic heating with  $\beta = -1$ , the functional equation is

$$\mathcal{L}.F = \lambda \left[ (1 - S_2)^N - \frac{X_0^2 - X^2}{X_0^2} \frac{3\mu_0}{8\pi} (1 - S_2)^{N-1} \frac{dS_2}{dF} \cdot \left( \frac{dS_2}{dF} \right)^{-1} \right]$$

with  $F = \begin{cases} 0 & \text{on the boundary} \\ 1 & \text{on the magnetic axis} \end{cases}$

$$S_2 = \iint \frac{X_0}{X} ds \neq S$$

$\lambda$  is an eigenvalue,  $N = \frac{3}{2(\alpha+1)}$ ;  $\mu_0$  is a factor of the order of some units

giving the difference between the real transport coefficient on the magnetic axis and  $K_R$ .

These equations have been studied. Analytical and numerical results for different cross-sections have been obtained (ellipse, racetrack, D).

Nonlinear Drift Tearing Modes<sup>\*</sup>

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A previously described 2½ dimensional code,<sup>1</sup> developed for the treatment of nonlinear tearing modes, has been modified to include terms considered to be important for typical present day tokamaks. Specifically, these additions take into account the Hall effect, magnetic viscosity, and parallel compression. The effect of these terms on the linear growth rate, nonlinear saturation and energy balance will be discussed, as well as an interpretation of the rotation of the magnetic islands seen experimentally.

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<sup>\*</sup>Work supported by U. S. ERDA Contract No. E(11-1)-3073.

<sup>†</sup>Permanent address Oak Ridge National Laboratory, Oak Ridge, Tennessee.

<sup>1</sup>Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion, Berchtesgaden W. Germany.

# TIME EVOLUTION OF FLUX-CONSERVING EQUILIBRIA<sup>1,2</sup> WITH NONCIRCULAR CROSS SECTIONS IN A D-T TOKAMAK\*

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In a D-T tokamak, auxiliary heating of an ohmic plasma toward an ignited plasma can be sufficiently rapid that magnetic flux and safety factor  $q(\psi)$  are well conserved within the plasma. The equilibrium evolution is then determined by the pressure evolution  $p(\psi, t)$ , which in turn can be obtained by solving the flux surface-averaged energy balance equations. In a one-fluid model with the heat conduction coefficient  $\chi_T(\psi) \propto T^\delta/n^\gamma$ , the dependence of the equilibrium evolution on temperature density and auxiliary heating power deposition profiles is investigated for D-shaped, elliptic and circular plasma cross sections.

For example, consider a D-shaped plasma with  $B_T = 43$  kG,  $R_O = 5$  m,  $a = 1.25$  m and  $b = 2.00$  m. Such a plasma with peaked density and temperature profiles (so that  $q_a/q_o = 4$ ), heated to  $\bar{\beta} = 3\%$  at  $\bar{n} = 7 \times 10^{13} \text{ cm}^{-3}$ , will result in a D-T  $\alpha$ -power of 40 MW. This  $\alpha$ -power is concentrated near the magnetic axis with a half-height width of 1.10 m, permitting reduction of the required depth of auxiliary heating by a factor of 2 to 3. With  $\gamma = 1$ ,  $\delta = 0.5$ , and  $\tau_E \approx 1\text{-}2$  sec, and assuming injection increases  $\bar{n}$  from  $5.6$  to  $8.5 \times 10^{13} \text{ cm}^{-3}$ , a net auxiliary heat power of 50 MW would bring  $\bar{\beta}$  from 0.7% to 6.0% in 3 sec. After this,  $\bar{\beta}$  would stabilize at 8.8% with the  $\alpha$ -power increasing from 110 MW to 140 MW. These results and others will be presented. Since  $\psi$  is used as an independent variable in the formulas, generalizations will also be discussed to include the flux diffusion and the particle balances in the light of Pao's formulations.<sup>3</sup>

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\*Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>1</sup>J. F. Clarke and D. J. Sigmar, Phys. Rev. Lett. 38, (1977).

<sup>2</sup>R. A. Dory and Y-K. M. Peng, Nucl. Fusion 19, 21 (1977).

<sup>3</sup>Young-Ping Pao, Phys. Fluids 19, 1177 (1976).

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The classical method<sup>1</sup> of determining the stability of MHD equilibria against small perturbations is generalized to the case of an electrically resistive and thermally conductive imploding dynamic pinch so that the onset and linear behavior of three dimensional non-equilibrium instabilities can be studied using 2-D codes. The formulation, a logical extension of linear equilibrium stability theory, describes the 3-D motion of a dynamic plasma as a perturbation of its 2-D motion. All of the fluid variables are expressed as  $Q(\vec{x}, t) = Q_0(x_1, x_2, t) + \epsilon Q_1(x_1, x_2, x_3, t)$ , where  $\epsilon$  is an arbitrarily small parameter. Substituting this form into the single fluid scalar resistivity and thermal conductivity MHD equations, one expands in powers of  $\epsilon$  keeping only zeroth and first order terms. The zeroth order equations are the usual set and give the 2-D axisymmetric motion<sup>2</sup>. The first order equations are linear in the first order quantities, with the coefficients depending on the zeroth order quantities. This linear problem for the first order variables can be Fourier analyzed in the ignorable coordinate of the 2-D motion, resulting in a completely 2-D problem and considerable savings in data storage and computation time. The non-linear 3-D motion is sacrificed to achieve this savings, but if by the method given here the pinch is found to be stable throughout its time history, then such 3-D motion does not occur. At intervals in the time history of the pinch, the least stable perturbation mode is calculated by "freezing" the zeroth order variables and advancing the first order quantities as if the zeroth order ones were constant in time, until the fastest growing mode swamps all the others. The freezing of the zeroth order quantities is permissible because the initial conditions of the first order variables are arbitrarily prescribable. This technique turns on the perturbation at the time of interest and discovers by iteration those initial conditions which result in the fastest growth in the succeeding time step. To test this idea, the problem is reduced one further dimension to a cylindrically symmetric straight pinch. Previously,<sup>3</sup> a study of sausage modes was reported. Here, dynamically induced Rayleigh-Taylor instabilities are shown to appear in a theta pinch at just those times as given by theory and experiment and with the correct growth rate.

1. I.B. Bernstein et al., Proc. Roy. Soc. A244, 17 (1958)
2. H.C. Lui and C.K. Chu, Phys. Fluids 18, No. 10, 1277 (1975)
3. M.F. Reusch, Paper presented at A.P.S. Meeting San Francisco 1976

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LINEAR AND NON-LINEAR NUMERICAL STUDIES OF RESISTIVE  
INSTABILITIES IN CYLINDRICAL GEOMETRY

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We use linear and non-linear resistive MHD computer codes to study resistive instabilities in cylindrical geometry. No special ordering of terms is assumed. In the non-linear regime the three dimensional problem is reduced to two dimensions by assuming helical symmetry, and the resulting equations are solved on an Eulerian mesh using implicit methods. Magnetic field configuration which are appropriate to FCT and reversed field pinches are investigated.

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## The Evolution to Equilibrium of Helically Symmetric Theta Pinches\*

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The time-dependent, single-fluid, dissipative MHD equations are solved for a straight theta pinch geometry with additional  $\ell=1$  helical fields. The previously developed two-dimensional code<sup>1</sup> is adapted into  $r$  and  $\varphi$  coordinates,  $\varphi$  being the helical coordinate  $\theta - kz$ . The model and boundary conditions are basically similar to those introduced by Weitzner<sup>2</sup>. The problem is attacked in two different ways. In a first approach the plasma is initially homogeneous and its motion is followed beyond the field rise time, until it nearly reaches equilibrium. In the second one the calculation starts with a plasma in a rigid rotor equilibrium, after the initial implosion phase is completed. The helical fields are then applied, and the new equilibrium position of the plasma is obtained. In all cases the transverse helical shift of the plasma is found to oscillate.

The results of this diffuse profile model are compared with the sharp boundary helical equilibrium theory<sup>3</sup>. The three parameters on the right-hand side of the expression for the helical shift<sup>3</sup>:  $\delta_{1a} = \frac{B_{hel}/B_0}{k(1-\beta/2)}$ , are varied, and for all values there is agreement within 10% between the two models, provided that the compression coil is far from the plasma column. There is less agreement, however, when the compression coil is close to the vacuum vessel, the diffuse profile calculation yielding then a shift larger than what sharp boundary theory predicts.

\* Work supported by U.S. ERDA contract EY-76-S-02-2456.

<sup>1</sup> H.C. Lui and C.K. Chu, Phys. Fluids 18, 1277 (1975).

<sup>2</sup> H. Weitzner, Los Alamos Report, LA-4599-MS (1971).

<sup>3</sup> A. Blank, H. Grad and H. Weitzner, IAEA (Vienna), vol.11, p. 607 (1969).

# PERTURBED MAGNETIC FIELD PHASE SLIP FOR TOKAMAKS<sup>\*</sup>

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Given a set of rotating helical current filaments within a cylindrical plasma and a set of fixed magnetic field detectors at the edge of the plasma, a mathematical model is used to investigate the observable effect of rotation or longitudinal motion in the intervening plasma. If the soft X-ray signals from magnetic islands indicate the instantaneous position of the helical current filaments within the plasma, then the phase difference between the X-ray signal and the magnetic fluctuations should provide a diagnostic of tokamak plasma rotation. The relative motion between the islands and the surrounding plasma is measured by the "slip"  $S(r) = \omega - kv_z(r) - mv_\theta(r)/r$ , where  $\omega$  is the rotation frequency of the islands and  $v$  is the plasma velocity.

The plasma velocity is added to Rosenbluth's reduced equations<sup>1</sup> to compute the phase difference between the island position and the perturbed magnetic field at the edge of the plasma as a function of the slip for a variety of resistivity profiles. The current profile within the islands is taken from nonlinear computer results.<sup>1</sup> For peaked conductivity profiles, the phase difference is observed to saturate and then decrease at large values of the slip.

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<sup>\*</sup>Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

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<sup>1</sup>R. B. White, D. Monticello, M. N. Rosenbluth and B. V. Waddell, to appear in Phys. Fluids.

## PERTURBED MAGNETIC FIELD PHASE SLIP FOR TOKAMAKS

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For the  $m = 2$  internal instability in ORMAK, experimental measurements of the phase difference between the X-ray PIN diodes (sensitive to temperature fluctuations and, therefore, to current perturbations localized around the  $q = 2$  surface) and the magnetic loops (located near the conducting shell) have been made and will be analyzed shortly. A simple model is investigated so as to calculate this phase difference. The model is a generalization of Rosenbluth's reduced resistive MHD equations<sup>1</sup> by allowing incompressible equilibrium flows. The plasma is assumed to be helically symmetric and extending to a perfectly conducting wall in a cylindrical geometry. Basically, one considers the effects of fluid rotation on the magnetic fields produced by the saturated (unrotated)  $m = 2$  magnetic island current as found from the non-linear MHD codes.<sup>1</sup> The dependence of this phase difference on the slip between the  $m = 2$  frequency of the signals and the plasma rotational frequency, and on the scanning location of the X-ray diode will be presented for various ORMAK conductivities.

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Work supported by U.S. ERDA Contract E(40-1)-5260 and Union Carbide for ERDA.

\* Department of Electrical Engineering, University of Wisconsin-Madison.

<sup>1</sup>R. B. White et al. Paper P6, Sherwood Meeting 1976, Madison, Wisconsin.

POSTER SESSION B

## ELECTRICAL CONDUCTIVITY OF TOKAMAK PLASMAS CALCULATED BY CPM CODES\*

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We have used the Collisional Plasma Model (CPM) to calculate the parallel conductivity for several values of collisionality,  $Z_{\text{eff}}$  and aspect ratio, using parameters characteristic of present experiments (e.g. ORMAK) as well as in the slideaway regime. The accurate treatment of the Fokker-Planck Collision operator in combination with toroidal trapping effects, in the regime of finite values of  $E/E_{\text{DREICER}}$ , removes the ambiguities introduced by the ad-hoc model treatment of this problem by Hui, Windsor, and Coppi.<sup>+</sup>

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\*Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>+</sup>B. H. Hui, N. Windsor, and B. Coppi, "Collisional Theory of Electrical Resistivity in Trapped Electron Regimes" NRL Memorandum Report 3349.

# Ion Cyclotron Instability Driven by Neutral Beam Injection\*

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## Abstract

Ion cyclotron waves and their harmonics have been observed predominantly in Q-machines during injection of energetic ions into low temperature plasmas. We consider whether such modes can be excited in current and future large tokamaks such as TFR and TFTR. We examine the stability of the extraordinary ion cyclotron wave and its harmonics in the fast and slow wave limits by calculating the energy transfer from ion beams (with transient Coulomb collisional distribution function from a monoenergetic source) to the wave. Critical density ratios,  $(n_b/n_e)_{crit}$ , and growth rates,  $\gamma/\omega_{ci}$ , are obtained from these calculations and used to determine the susceptibility of the wave to destabilization as a function of injection energy and angle with respect to the magnetic field.

We show that unlike the low temperature experiments for parameters representative of TFR and TFTR the fast wave limit of the fundamental mode is not accessible. The slow wave limit is, but due to the larger  $k_{||}$ 's, appears to be stable except under extreme conditions. In contrast, the harmonics of this wave, ie  $\ell = 2$  and  $\ell = 3$ , are shown to have low critical thresholds particularly for D-T systems. These thresholds appear to have little dependence on injection angle and decrease as the injection energy is increased. In all cases, these modes are extremely sensitive to the ion temperature of the background plasma and can be stabilized by sufficiently high temperatures.

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## A Unified Slowing-Down Theory for Fusion Plasmas\*

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An accurate treatment of charged particles slowing down in magnetically confined fusion plasmas is important to fusion applications in three areas: neutral-beam heating; two-component fusion; and ignition via fusion product heating. Slowing down rates and times for charged particles in the D-T and D-<sup>3</sup>He plasmas are calculated in the framework of a unified theory, cf. original work by Hubbard<sup>(1)</sup>, and Kihara and Aono<sup>(2)</sup>. In contrast to conventional treatments, this method includes both collective effects as described by the Balescu-Lenard kinetic equation and effects due to close impact collisions such that "divergences" of the Coulomb logarithm vanish.

Numerical results are presented to compare the unified theory with other well-known treatments of charged-particle slowing down in the plasma regimes of mirrors and Tokamaks. For plasma conditions typical of both mirror and Tokamak devices, the energy loss rates for the unified theory are 5 to 20% higher than those of Fokker-Planck calculations, resulting in shorter slowing down times. While this is not a large discrepancy, use of the present formalism has some advantages. It does not involve a greater complexity of computational methods than Fokker-Planck techniques; yet a much wider range of validity is offered. Consequently, the unified scheme is potentially an important and convenient method for fusion calculations.

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\*Partly supported by U. S. ERDA.

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## ANISOTROPY EFFECTS ON THE CYCLOTRON RADIATION EMITTED BY TOKAMAK PLASMAS\*

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When propagation perpendicular to the magnetic field is considered, the generation and propagation of cyclotron radiation in tokamak plasmas is asymmetric with respect to the minor axis. The temperature and density profiles are symmetric, but the toroidal field introduces the asymmetry. Radiation of a given frequency is generated and absorbed at surfaces of constant magnetic field. The power emitted or absorbed is a function of temperature and density, which are not constant on these surfaces. Therefore the power generated at a given frequency has a strong spatial dependence. In propagating, the radiation is subject to plasma dispersive effects. The extraordinary mode is absorbed at the upper hybrid resonance and reflected at the upper hybrid cutoff. Ordinary mode radiation is reflected from the surface  $\omega = \omega_{pe}$ , and evanescent when  $\omega < \omega_{pe}$ . The geometry of the surfaces of generation, absorption, and reflection are complicated by the asymmetry of the toroidal field. Thus the radiation propagates in an anisotropic medium due to the complex geometry of the absorbing and reflecting surfaces. We have studied these effects on the cyclotron radiation generated and transported along lines perpendicular to the toroidal field and passing through the center of the current channel.

\*Work supported by NSF and the University of Maryland Computer Science Center.

# The First Fully Implicit 2-D Fokker Planck Solver Using ICCG Algorithm\*

by

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## ABSTRACT

The standard numerical methods (ADI and splitting) for advancing the 2D Fokker-Planck equation have been hampered by the fact the methods are not fully implicit. We have solved the 9-point Fokker-Planck difference equations fully implicitly via a non-symmetric Incomplete Cholesky-Conjugate Gradient algorithm (ICCG) (David S. Kershaw, "The Incomplete Cholesky-Conjugate Gradient Method for the Iterative Solution of Systems of Linear Equations", Preprint UCRL-78333). A considerable improvement (a factor of ~5) in computer running time has been obtained. The reasons for this improvement will be discussed.

\*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

Self-Consistent Calculation of Oxygen Impurity Diffusion in  
Tokamak Plasmas\*

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Abstract

A transport code has been developed which follows the time-dependence of the radial profiles of electron density, ion density, electron temperature, ion temperature, poloidal magnetic field, and the densities of 5 different ionization states of oxygen,  $O^{+4} - O^{+8}$ . (Coronal equilibrium is only assumed for  $O^{+1} - O^{+3}$ .) Ionization and recombination terms are included in the diffusion equations for the impurities. Boundary conditions giving either the particle densities or particle fluxes may be used. The implicit predictor-corrector code uses a finite-element method with 13 cubic spline trial functions on a nonuniform partition. Neoclassical transport coefficients for a 6-ion species plasma are calculated in kinetic theory subroutines, which use a generalization of the method of Chu, Rawls, Wong and Hinton (Phys. Fluids 20, 127, 1977). The ions and impurities are assumed to be in the range of collisionalities from the plateau regime to the weakly-equilibrated Pfirsch-Schluter regime. Preliminary results will be given.

\*This work supported by U.S. ERDA Contract EY-76-C-05-4478.

## A Numerical Model of ALCATOR\*

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In a number of tokamak experiments, the plasma density can be increased by bleeding gas into the vacuum chamber during the discharge. For example, in ALCATOR the density can be varied by an order of magnitude while in PLT there is a more modest change  $\sim 2$ . In both PLT and ALCATOR, the energy confinement is improved as the density is increased. Further, in both of these experiments the density increases without any apparent inversion of the density profile.

A one-dimensional radial transport code is used to study the density build-up in ALCATOR. The particle and heat transport contain contributions from the three regimes of neo-classical theory; in addition, we assume anomalous particle diffusion and electron heat transport due to current-driven drift waves.<sup>1</sup> Ionization source terms are calculated using a fast Monte Carlo scheme.<sup>2</sup>

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\*Work supported by U. S. ERDA Contract No. E(11-1)-3073.

<sup>†</sup>Visiting scholar from Culham Laboratory

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2. M. H. Hughes and D. E. Post, Princeton Plasma Physics Lab. Report PPL-1335 (1977)

A Monte Carlo Algorithm for Calculating  
Neutral Gas Transport in Tokamaks<sup>\*</sup>

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Monte Carlo techniques have been used to calculate neutral gas distributions in tokamaks. The algorithm uses track length estimators, suppression of absorption, and splitting with Russian roulette to reduce the variance, so that the algorithm is economic. The resultant package is small in memory requirements and very fast (~ 15 seconds of PDP-10 KI time per neutral density profile). The tokamak is modeled as a three-dimensional cylinder, and the plasma parameters are specified as a function of the radial coordinate of the cylinder. The plasma ions are assumed to be Maxwellian, and an arbitrarily large number of charge-exchange generations can be followed. The effects of wall reflection and sputtering yields can be easily computed within the model. Wall reflection of neutrals was not found to have a large effect on neutral density distributions for typical tokamaks. The algorithm has been incorporated in a one-dimensional transport code. The code has also been used to predict energy spectra of charge-exchange neutrals, which form the basis for measurements of ion temperatures in tokamaks.

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<sup>\*</sup>Work supported by United States Energy Research and Development Administration Contract No. E(11-1)-3073.

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# A TECHNIQUE FOR MONTE CARLO SIMULATION OF VELOCITY DIFFUSION IN TWO DIMENSIONS\*

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## ABSTRACT

A technique for Monte Carlo simulation of velocity diffusion in two dimensions with a rotationally symmetric but otherwise arbitrary diffusion tensor,  $\underline{D}_{vv}(|\underline{v}|)$ , is presented. Such a scheme is useful for including in particle codes diffusion arising from microinstabilities not simulated by the code.

In such a scheme, the Green's function for the appropriate diffusion equation is used as a probability distribution from which random velocity increments are chosen each time step. An approximate expression for the Green's function valid for times less than the diffusion time,  $\tau_D \sim v^2 / \text{Tr}\{\underline{D}_{vv}\}$ , is constructed from its three lowest order moments.

The method is used to consider the special case of DCLC turbulence, with  $\underline{D}_{vv}$  derived from the quasilinear theory of Baldwin, Berk and Pearlstein<sup>1</sup>. The approximate Green's function obtained agrees well with a finite difference solution to the exact diffusion equation. The simulation scheme is then used to propagate a particular distribution of particles forward in time a total of 800 timesteps. The resulting distribution agrees to within statistical uncertainty with that obtained by a finite difference solution to the equation governing the diffusive evolution of the same initial distribution.

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\*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

<sup>1</sup>D. E. Baldwin, H. L. Berk and L. D. Pearlstein, Phys. Rev. Letts. 36, 1051 (1976).

MONTE CARLO SIMULATION OF A QUASILINEAR TRANSPORT  
MODEL FOR MIRROR PLASMAS\*

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ABSTRACT

A Monte Carlo version of the one dimensional quasilinear transport model of Berk and Stewart<sup>1</sup> has been constructed which includes neutral beam injection, electron drag, end loss, low energy stream and velocity diffusion induced by DCLC turbulence. The turbulence level is determined self-consistently as a function of time from the ion distribution function and the growth rate obtained from the linear dispersion relation.<sup>2</sup>

In general, the findings of Berk, Rognlien and Stewart<sup>3</sup> are qualitatively reproduced: depending on the value of a critical parameter, the turbulence level reaches either a state of marginal stability, or one with large amplitude oscillations (bursts).

The code is presently being used to study the effects of the acceleration of various rates in the Superlayer particle code.

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\*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

<sup>1</sup>H. L. Berk and J. J. Stewart, Lawrence Livermore Laboratory Report UCRL-78407 (1976). To be published in Phys. Fluids.

<sup>2</sup>D. E. Baldwin et. al., Proc. Fourth Conf. Plasma Phys. and Controlled Nuc. Research, Madison, Wisc., 1971 (I.A.E.A., Vienna, Austria, 1972) II, p. 375.

<sup>3</sup>H. L. Berk, T. D. Rognlien and J. J. Stewart, Lawrence Livermore Laboratory Report UCRL-79254 (1977)

## RECENT RESULTS OF TOKAMAK MODELING\*

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The two-dimensional tokamak code, G2M<sup>1</sup>, has been used to model the Alcator and PLT devices. The code solves the equations of conservation of particles, energy, and flux, employing various models for transport coefficients.

The Alcator study examined the scaling of energy confinement time with density. We find that at high densities the use of neo-classical transport is sufficient to obtain experimental agreement, while at lower densities anomalous transport due to drift waves must be invoked.

The PLT study examined the effect of impurities on the energy balance of this device. We find that with low levels of a high Z impurity, such as tungsten, there is a large amount of radiation which causes the central plasma temperature to drop considerably. Inclusion of a low Z impurity, such as oxygen, can increase the ohmic heating to a level where the ohmic heating dominates the impurity radiation.

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\* Work supported by the U. S. Energy Research and Development Administration.

1. R. N. Byrne and H. H. Klein, G2M, A Diffusion Time Scale Tokamak Code, submitted to J. Comp. Phys. (1976).

## INITIAL PHASE OF TOKAMAK DISCHARGES\*

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The initial phase of a tokamak discharge is simulated by a computer code which solves the density and temperature equations for the species  $H_1^0$ ,  $H_2^0$ ,  $H_1^+$ ,  $H_2^+$ , electrons, the ionization state equations for oxygen impurities, and either a global current or a radially dependent set of current density equations. The models include plasma, wall, and radiation physics. The equations also form the basis of a plasma boundary model for radially dependent simulation, including the processes of ionization, excitation, dissociation, charge exchange, recombination, energy-dependent neutral wall-reflection, diffusion, and refluxing, and time dependent oxygen ionization.

Agreement between the simulation and the early time behavior of an ORMAK discharge is obtained with a resistivity anomaly ( $\leq 3$ ). A comparison with the radially-dependent current-penetration model shows that a skin effect does not occur at early times ( $< 3$  msec) due to the low plasma temperatures. The temperature is initially held at  $\sim 4$  eV by excitation radiation from  $H_1^0$  during breakdown. At later times, both transport losses and impurity line radiation are significant. Of the 0.2 volt-seconds required to raise the plasma current to 100 KA in 6 msec, approximately half are calculated to be due to resistivity. This model is extended to the simulation of larger plasmas, and the role of impurities in current penetration is explored.

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## QUASILINEAR MODEL OF A MIRROR PLASMA WITH RADIAL TRANSPORT\*

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## ABSTRACT

We have developed a new computer code that includes radial transport to study a plasma build-up in a mirror machine dominated by drift cyclotron loss cone mode turbulence. Previous quasilinear calculations have treated turbulent diffusion in velocity space only, i.e. 1D in  $V_{\perp}$  or 2D in  $V_{\perp}$  and  $V_{\parallel}$ , ignoring radial dependence. It is expected, however, that effects of radial transport due to the turbulence is important in determining the design of future mirror machines.

We have derived quasilinear equations describing evolution of ion distribution functions in  $V_{\perp}$  and  $r$  and wave energy in  $r$ . Together with an equation for electron temperature, they constitute a model with which we can study turbulence and classical transport processes of a mirror plasma.

Initial results of simulation for 2XIIB indicate the following: (1) Marginal stability obtained in 1D and 2D simulations was not reached without a cold plasma stream. (2) A reasonable amount of stream plasma peaked near the edge of hot plasma can bring the system to marginal stability. (3) Ion temperature develops radial gradients as well as density gradients and this temperature evolution is an essential factor in the relaxation to a marginally stable state.

Further development of the code is being carried out to take into account the wave dispersion relation self-consistently.

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PROFILE EFFECTS ON PLASMA TRANSPORT IN TOKAMAK FUSION  
REACTORS UNDER THE INFLUENCE OF TRAPPED PARTICLE MODES<sup>\*</sup>

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Recent analytic work by Clarke<sup>1</sup> indicates that density driven micro instability modes may be of smaller consequence to high temperature tokamaks than had been previously supposed.<sup>2</sup> In substantiation of this numerical work was performed using a 1-D "multifluid" transport code including a divertor,<sup>3</sup> and several cold fueling profiles. The numerical work indicates that "edge" fueling even in the presence of a divertor may have added advantages. It will also be demonstrated that one need not have fresh fuel (D & T ions) injected into the center of a fusing plasma in order for it to maintain its thermal equilibrium.

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<sup>\*</sup>Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>1</sup>To be presented at this 1977 meeting.

<sup>2</sup>S. O. Dean, et al, "Status and Objectives of Tokamak Systems for Fusion Research," WASH-1295, (U.S.A.E.C.-1974).

<sup>3</sup>A.T. Mense, Ph.D. Thesis, Nuc. Engr. Dept., Univ. of Wisconsin, Madison, 1977.

FOKKER-PLANCK/TRANSPORT SIMULATIONS OF  
COUNTERSTREAMING ION PLASMAS IN THE PDX EXPERIMENT\*

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Tokamak plasmas in which the ion pressure greatly exceeds the electron pressure can be established by injecting all ions via energetic neutral beams, and by minimizing recycling of cold plasma and neutrals with a magnetic divertor. Some advantages of energetic-ion Tokamak plasmas<sup>1</sup> are that: (1) large plasma beta is attainable even with modest  $n_e \tau_e$ ; (2) steady-state operation is possible in principle; and (3) the fusion reactivity is large even with low-energy beam injection.

The PDX device, equipped with a poloidal magnetic divertor and up to 8 MW of 40 KeV neutral beams, is especially relevant for testing CIT (co- and counter-injection) energetic ion operation. Using a Fokker-Planck/radial transport code<sup>2</sup>, we have calculated the performance of deuterium CIT plasmas in PDX.

Results are presented of transport studies in which the injection energy, injection current, impurity content and neutral density are varied. In particular, for  $P_B = 8$  MW at 40 KeV,  $Z_{eff} = 1$ ,  $n_e \tau_e = 2 \times 10^{12}$  cm<sup>-3</sup>sec and the best present estimate of background neutral density,  $Q$  (scaled to D-T systems) equals 0.20, or  $6 \times 10^{17}$  neutrons/sec.

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2. A.A. Mirin, et. al., J. Comp. Phys. 23 (1977)

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## PARTICLE CONFINEMENT BY AMBIPOLAR POTENTIALS IN MIRROR MACHINES\*

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## ABSTRACT

We consider the basic problem of particle endloss in a magnetic mirror field with an electrostatic confining potential. We have generalized the analysis by Pastukhov [Nucl. Fusion 14, (1974),3] for end-loss of electrostatically confined electrons in a square-well mirror to apply to ions, multi-species plasmas, and more general magnetic field profiles. These generalizations allow application to tandem mirror systems, as described in another paper at this meeting. The resulting expressions are compared with studies made with one- and two-dimensional Fokker-Planck codes. Alpha particle trapping and thermalization in a tandem mirror reactor have also been analyzed with these Fokker-Planck codes. Results will be presented for the scaling of ion and electron confinement with mirror ratio and potential, and for alpha trapping and thermalization as a function of electron temperature.

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## Computer Model of Gas Feed Experiments in Mirror Machines\*

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### ABSTRACT

The build-up of plasma in the 2XIIB mirror machine to a moderately high beta using a gas feed box is simulated by a two-dimensional Fokker-Planck code together with an axial flow code. The Fokker-Planck code describes the evolution of the hot ion distribution arising from neutral beam injection, and the axial flow code describes the flow of cold plasma generated by the gas feed box. The cold plasma from the gas box stabilizes the drift-loss-cone mode which is included self-consistently in the Fokker-Planck code via quasilinear theory. The effect of a higher plasma beta is reflected in a spreading of the angle of neutral beam injection and an increase in the total mirror ratio. In addition, the corresponding higher plasma density results in the plasma no longer being thin to the neutral beams. We thus include a model where a significant portion of the beam can be absorbed. The code results are compared to experimental measurements of 2XIIB. The model does not pertain to the highest density regime of 2XIIB where the possibility of field reversal becomes an issue.

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NUMERICAL EVALUATION OF MULTI-IMPURITY PFIRSCH-SCHLÜTER TRANSPORT<sup>\*</sup>

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Using transport coefficients from a recent theory<sup>1</sup> of multiple species particle transport in the Pfirsch-Schlüter regime, particle and heat fluxes have been calculated for plasma parameters characteristic of the edge of the ORMAK plasma.<sup>2</sup> A computer code PSCOEF was written to evaluate the coefficients for an arbitrary number of plasma species, including electrons. The plasma was modeled using electrons, protons and several species each of oxygen and iron impurities.

We find that where the electron temperature gradient scale length is relatively small, such as near the plasma edge, atomic processes, predominately ionization, lead to strong density gradients for the different impurity ions. These strong density gradients can drive the diffusion of the impurity species rapidly enough to preclude the establishment of a corona equilibrium distribution, contrary to the situation if the gradients were only on the order of the other gradients of the problem.

When the electrons are equilibrated in temperature with the ions, ion heat conduction is reduced by approximately two orders of magnitude, i.e., a factor  $\sim (m_e/m_{ion})^{1/2}$ . This follows directly from theory. The direction of some of the ion particle fluxes can be reversed by ion-electron friction. Since the ion heat flow in the edge region could be dominated by convection, this flux reversal could have a significant effect on the ion heat flow.

We are currently coupling PSCOEF to a tokamak transport code.<sup>3</sup>

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<sup>\*</sup>Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

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<sup>1</sup>S. P. Hirshman, Princeton Plasma Physics Laboratory Report PPL-1291 (1976).

<sup>2</sup>M. Murakami and R. C. Isler, private communication (1977).

<sup>3</sup>T. Amano and E. C. Crume, paper submitted for this meeting.

# ENERGY BALANCE AND IMPURITIES IN THE SIMULATION CODE MAKOKOT

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The Fontenay-aux-Roses 1 D toroidal transport code /1/, integrates the diffusion equations for current density, electron particle density, ion and electron temperatures. The neutral evolution is represented by a Boltzmann equation. We have also the possibility of treating four impurity species : Carbon, oxygen, iron and molybdenum. Our model includes the physical atomic data reported in /2/, with in particular dielectronic recombination coefficients.

We present a few examples of applications of this code. The first is a study of the saw-tooth oscillation of the soft X-ray signal. This signal is proportional to the population of electrons whose energy is  $\gtrsim 3$  keV. We find that it is possible to fit the experimental results by multiplying  $\chi_e$  (heat conduction of the whole electron population) during a short period at the exterior to the surface  $q = 1$ .

We have also studied the energy balance of a TFR discharge /3/. The numerical results that we shall present fit well the experimental data. They have been obtained by assuming : for the electron particle diffusion and the heat electron conduction an anomalous form of about 2000 times the Pfirsch-Schluter expression, for the heat ion conduction a neoclassical form and that the neutrals are emitted by the walls with energies of order of 100 eV.

Finally we have simulated the impurity behaviour in TFR discharges. We find that, for representing the experimental behaviour it is necessary to add to the neoclassical flux an anomalous flux, linked (for ambipolarity reasons) to the anomalous electrons flux  $\phi_e$ . Although it is possible to estimate the magnitude of  $\phi_e$  from the knowledge of the neutral density, the nature of the anomalous electron diffusion is not known, so we hope that the impurity behaviour can give some light on this field. We have found that we can obtain a good representation of the injection of an oxygen puff in TFR, by adding to the neoclassical flux an anomalous flux, proportional to the impurity density gradient.

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A Comment on The Numerical Solution of Neoclassical  
Transport Equations\*

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Abstract

In the numerical solution of neoclassical transport equations difficulties have often been encountered with zero boundary conditions, which has required the use of pedestals or the addition of other diffusion terms. This problem has been studied by means of a model problem consisting of two coupled parabolic equations for the time evolution of the pressure and density. The driving forces are the logarithmic derivatives of the density and temperature. It is noted that neither the model system nor the neoclassical system satisfy the ellipticity condition usually assumed for typical existence theorems dealing with parabolic systems. This failure, which is associated with the off diagonal transport terms, is conjectured to imply that there are certain initial conditions for which no solution exists. A linear analysis shows that below a critical boundary temperature the solution becomes negative, i.e. unphysical, after a certain time. Results of both a linear and a nonlinear analysis will be given.

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POSTER SESSION C

LINEAR AND NONLINEAR PROPERTIES OF THE M=2 TEARING MODE  
IN CYLINDRICAL PLASMAS

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Previous investigations of the  $m=2$  tearing mode in the low- $\beta$  cylindrical plasma approximation<sup>1)</sup> have been extended in several directions. The linearized equations have been solved to obtain the (complex) growth rates and eigenfunctions taking into account diamagnetic drifts and viscosity. A critical value  $\omega_c^*$  is found, corresponding roughly to  $\omega^* \sim \gamma_T$ . For  $\omega^* < \omega_c^*$  mode structure and growth rate rapidly approach the asymptotic limit  $\omega^* \rightarrow 0$ . Small scale drift waves appear as  $\omega^*$  approaches  $\omega_c^*$  but remain localized around the critical surface  $r_s$ . For  $\omega^* > \omega_c^*$  which is the case for tokamak plasmas, these drift waves propagate across the entire system. Starting with some initial perturbation many exponentation times are required for the mode struction to become stationary. The magnetic perturbation  $\psi$  is relatively unaffected. The growth rate suffers a sharp drop (by a factor of  $\sim 10$ ) for  $\omega^*$  exceeding  $\omega_c^*$ . Introducing a realistic amount of viscosity strongly reduces the small scale drift wave amplitudes without changing  $\gamma$  significantly.

The nonlinear behavior, in particular the saturation level, is found to depend on the resistivity  $\eta(\psi)$  in the islands. Since  $\partial j / \partial \psi > 0$  within the islands is found numerically, independent of the assumptions on  $\eta$ ,  $\partial \eta / \partial \psi < 0$  within the islands is required for a stationary state.

The nonlinear programme has been extended to include drift effects and viscosity. The main purpose is to study the nonlinear behavior in the drift wave regime  $\omega^* > \omega_c^*$ . Furthermore a three-dimensional programme has been developed to investigate the coupling of the  $m=2$  mode to the  $m=1$  mode. First numerical results will be presented.

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# STABILITY PROPERTIES OF AN ANISOTROPIC GUIDING CENTER PLASMA AND RELATION WITH THE SUYDAM FUNCTION<sup>†</sup>

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This paper examines (a) the effect of energy anisotropy on the growth rate of internal kink modes in an unstable Guiding Center Plasma (GCP), and (b) the relation of the Suydam function to associated stability properties. The quantity  $\alpha = P_{\parallel} / P_{\perp}$  is an attractive parameter for this study, since (for given field profiles) the Suydam function  $S$  and the growth rate  $\gamma$  are relatively independent of other plasma parameters (e.g., beta). Assuming  $\alpha = \text{const.}$  and straight cylindrical geometry, we present a detailed parametric study of GCP equilibrium and stability properties for typical tokamak field profiles. The relevance of the Suydam function  $S$  is investigated by correlating the maximum growth rates with the magnitude of  $S$ , and by examining the ratio of consecutive eigenvalues for each set of system parameters. Except for a narrow range of  $\alpha$ , the numerical studies show that the maximum growth rate is a monotonically decreasing function of  $S$ . Also, the ratio of consecutive eigenvalues remains approximately constant, thereby making feasible the idea that the Suydam analysis can be extended to the region of maximum growth rate. A detailed examination of  $S$  reveals that shear is always stabilizing, although the degree of stabilization decreases as  $\alpha$  increases. Moreover, the study shows that the axial current contribution to  $S$  is destabilizing (stabilizing) when  $\alpha > 1$  ( $\alpha < 1$ ). In conclusion, even though the function  $S$  originally arose in studies of marginal stability, the present work shows that  $S$  is intimately connected with the growth rate in an unstable GCP, and that the Suydam analysis can be extended to the region of maximum growth. It is expected that the Suydam function has a similar relevance in an ideal MHD model.

<sup>†</sup>Research supported by the Energy Research and Development Administration.

\* On leave of absence from the University of Maryland, College Park, Md.

## DIFFUSE TOROIDAL HELICAL EQUILIBRIA\*

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Diffuse stellarator and Scyllac-like equilibria are sought as solutions to a set of two equations involving a potential-like function  $\phi$  ( $\underline{B} = \nabla\phi/|B|$ ), the kinetic to magnetic pressure ratio  $\beta(r,\theta)$  and an arbitrary function  $g(\beta)$ .<sup>(1)</sup> The correspondance of the present formulation to earlier results for toroidal skin current equilibria is demonstrated in the low  $\beta$  limit. In the high  $\beta$  limit, a large class of equilibria is obtained by adopting a small expansion parameter essentially proportional to  $(1 - \langle\beta\rangle)^2$ ; here  $\langle\beta\rangle$  is the  $\beta$  averaged over the plasma section.

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1) R. Y. Dagazian and J. P. Freidberg, Theoretical CTR, Wisconsin, 2C-6, (1976).

\*Work done under the auspices of U.S.ERDA.

## NONLINEAR EFFECTS ON FREE BOUNDARY IN SLAB GEOMETRY\*

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It is well established that the plasma semi-infinite slab cannot support any extensive modes or instabilities in the framework of linear infinite conductivity MHD theory.<sup>1</sup> On the other hand, it has been shown that an extensive nonlinear convective solution is possible in the absence of shear.<sup>2</sup> Here we investigate the effect of nonlinearity introduced by convective motion on the stability of an extensive free boundary marginal mode. This mode possesses a smooth enough profile to form the basis of a time-dependent small amplitude nonlinear analysis. Analogues from hydrodynamic theory are carried over.

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<sup>1</sup>G. P. Goedbloed and R. Y. Dagazian, Phys. Rev. 4 4 (1971).

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\*Work performed under the auspices of U.S.ERDA.

APPLICATION OF IDEAL MHD STABILITY CODE ERATO  
TO HIGH BETA TOKAMAK EQUILIBRIA\*

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The hybrid finite-element energy principle code<sup>1</sup> of CRPP has been used for an initial survey of the ideal MHD stability of the ORNL high beta Tokamak equilibria found<sup>2a</sup> by applying the flux conserving tokamak model<sup>2b</sup> for a D-shaped plasma with height/width of 1.7. When the conducting shell is removed to infinity, a beta limit of 1-2% is found. The modes are basically kink modes, but near threshold appear as rather localized ballooning modes. When the conducting shell is "near" the plasma, the threshold beta value is 3-4% for an unoptimized case. Future study is needed of optimization vis a vis cross-section shaping and control of the plasma pressure profile width.

Credible and encouraging (but not yet validated) results for a conducting shell at finite distance suggest that "near" the plasma means that the shell radius should be no more than 1.1-1.2 times the plasma radius. Such a configuration may be practical even in a reactor since the time scale on which the first wall needs to appear as a good conductor is relatively short. Conversely, if such a shell cannot be simulated, the ideal MHD model says that the equilibria now known will be unstable above the 2% beta level.

Further application of the ERATO code is in progress at ORNL for FCT equilibria and at CRPP for the design of the Variable Configuration Tokamak (TCV) in which it is hoped to test cross-section shaping.

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\* Research sponsored by the Fonds National Suisse de la Recherche Scientifique and by the U.S. Energy Research and Development Administration under contract with Union Carbide Corporation.

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Analytical Study of the Dynamical Development of Tokamak  
Equilibria in the Drift-Kinetic Approximation.\*

J.L. Fisher and J.E. McCune, M.I.T.--Analysis of the time development of finite beta Tokamak equilibrium is carried out in the banana as well as Pfirsch-Schluter regimes. Use is made of a drift-kinetic equation derived in the MHD ordering; i.e., bulk plasma flows are allowed to be as large as the order of the ion thermal speed. Thus the ambipolar electric field and associated flows are self-consistently generated. Significant flows in the developing discharge are found to modify neoclassical transport. In particular, the existence of a large class of stagnant particles with zero poloidal velocity, and which therefore do not experience the B-field modulation, is seen to modify low-collisionality transport. Results for the high collisionality regime are compared with an earlier MHD analysis<sup>1</sup> of a similar problem.

\*Supported by US ERDA Contract E(11-1)-3070

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# Non-Resistive Changes in Field Topology<sup>†</sup>

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It is frequently stated (and sometimes proved) that the magnetic field line topology is invariant under ideal MHD equations of motion. A theory of generalized adiabatic motion of systems with islands has been given<sup>1</sup> in which the topology can change. In this theory although dissipation is not overtly present, it is hidden in jump conditions across the separatrix. These features are also present in the classical theory of MHD shocks. As an example we provide an explicit description of field line reconnection near an X-point which involves the usual building blocks: constant states, simple waves, and shock waves. (This last problem has been repeatedly attacked by astrophysicists.<sup>2</sup>)

We can present a more surprising example of ideal MHD flow without shocks and with flow across a separatrix, breaking and reconnecting lines with no dissipation at all. Similarly, in applying  $\delta W$ , we can present a square integrable displacement,  $\int \rho_0 \xi^2 < \infty$ , in which the field line topology changes. The consequences with regard to stability of Doublet, etc. have to be investigated.

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<sup>†</sup> Work supported by U.S. ERDA Contract EY-76-C-02-3077.

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# MAGNETOHYDRODYNAMIC INSTABILITY OF STRAIGHT AXIS MIRROR QUADRUPOLE WELLS WITH STEEP RADIAL PRESSURE GRADIENTS\*

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## ABSTRACT

Conventional design criteria for shallow-radial-well large-aspect-ratio minimum-B mirror wells of Yin-yang or quadrupole symmetry assume that if good curvature is maintained everywhere (at finite  $\beta$ ) and if the plasma is carefully injected so as to maintain stability against local mirror or fire-hose modes, MHD stability of the system can be maintained to large  $\beta$  values, of the order of unity. This can be true provided very gentle radial pressure gradients are maintained. However, many current projected designs<sup>1</sup> which attempt to relegate the bulk of the pressure gradient to a boundary region--either for optimal filling of the well or to satisfy microstability criteria in the main body of the plasma--are unstable (in the guiding-center MHD limit) when  $\beta/\epsilon$  exceeds a number of order unity, where  $\epsilon$  is the thickness of the boundary layer divided by its radius. The source of the instability--which exists even when the curvature in the direction of the pressure gradients is everywhere good--is in the shear which is associated with the presence of a component of the curvature vector lying in the omnigenous drift surfaces.<sup>2</sup> We interpret the instability as the tendency of the plasma to circularize or straighten out the net (diamagnetic) current carrying path within the plasma which in the unperturbed equilibrium flows in the image of a baseball seam.

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## KINETIC THEORY OF BALLOONING MODES IN TOKAMAKS\*

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Recent numerical computations of the MHD instability of high- $\beta$  tokamaks have shown that ballooning-like modes appear to be the most dangerous instability in limiting the achievable  $\beta$  of the plasma.<sup>1,2</sup> In this paper, a unified, kinetic theory for ballooning modes and trapped particle modes in tokamaks is presented, including the effects of toroidal curvature drifts, trapped particles, finite Larmor radius, finite banana size, and Landau resonance. A matrix dispersion relation is derived and analyzed. The MHD ballooning mode and the trapped particles instabilities are shown to be the special cases. The FLR (drift) term is shown to have strong stabilizing effect on the usual MHD ballooning modes with poloidal mode number  $m > 1$ . The consequence on the  $\beta$ -limit of the high- $\beta$  tokamak is discussed.

\*Research supported by the Energy Research and Development Administration.

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Stabilization of Axisymmetric Instabilities  
in Noncircular Cross Section Tokamaks  
by Passive Feedback<sup>\*</sup>

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It has been reported previously<sup>1</sup> that numerically generated noncircular cross section "equilibrium" configurations, typical of the Princeton Poloidal Divertor Experiment (PDX), are unstable to a rapidly growing axisymmetric MHD instability which consists of both a vertical displacement and a complicated change of shape of the plasma cross section.

Here we extend and refine the previous study by considering the interaction between the field shaping coils and the plasma. This provides a passive stabilization mechanism, the strength of which is determined by the plasma coil mutual inductances and by the circuit equations for the coils. For realistic parameters, this feedback is strong enough to stabilize the axisymmetric instability in the PDX device.

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<sup>\*</sup>Work supported by U. S. ERDA Contract No. E(11-1)-3073.

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## Collisional Effects for Current Driven Drift Waves\*

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Anomalous transport via current driven drift wave turbulence in ALCATOR is of interest especially as a process possibly involved near the plasma edge when the density builds up during gas puffing.<sup>1</sup> Considerations of the stability and growth rates of such modes are, therefore, of interest.

The linear electron response to low frequency oscillations including the effects of Lorentz (angle scattering) collisions is calculated. A slab model is assumed with the background electron distribution function including the electron current taken to be of the Lorentz (Spitzer-Härm) type. Trapped particles are neglected for  $v_{*e} \gg 1$ . Conclusions concerning anomalous transport resulting from a current driven drift wave instability are drawn from numerical and analytic calculations for growth rates given by the model.

\* Work supported by U. S. ERDA Contract No. E(11-1)-3073.

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SEMI-COLLISIONAL  $m = 1$  INTERNAL KINK MODES\*

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A recent kinetic investigation of tearing modes<sup>1</sup> has shown that in present weakly collisional tokamak discharges electron thermal motion along B strongly modifies the structure and growth rate of tearing instabilities. Such effects also influence the internal kink mode. The analysis of this mode is complicated by the failure of the "constant  $\psi$ " approximation. Nevertheless, we find that when the MHD driving force is negligible, the regime of most interest in present discharges, previous theories of this instability are invalid when

$$\Sigma = \beta(\tau_R/\tau_H)^{2/3} (c^2/\omega_{pi}^2 r_s^2) > 1,$$

where  $\tau_R$  and  $\tau_H$  are the resistive and poloidal MHD time scales, respectively,  $\beta$  is the total plasma beta and  $r_s$  is the minor radius at the  $q = 1$  surface. The growth rate  $\gamma$  of  $m = 1$  mode becomes independent of the collision frequency when the above inequality is satisfied and, therefore,  $\gamma$  may not be reduced in the next generation of tokamak discharges. The inclusion of drift effects will also be discussed.

\*Work supported by ERDA and NSF

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## THREE DIMENSIONAL MHD TURBULENCE IN CYLINDRICAL FUSION PLASMAS\*

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Ideal MHD turbulence inside a rigid perfectly-conducting cylinder is treated by a truncated expansion of the fields in a set of vector eigenfunctions of the curl. The work is a direct generalization of that of Frisch et al<sup>1</sup> to cylindrical geometry. The eigenfunctions are solutions of the vector wave equation and are individually force-free, though the superpositions we make of them are not force-free. Expectation values of functions of the expansion coefficients are calculable from the canonical ensemble constructed from the three rugged invariants of the system: total energy, magnetic helicity, and cross helicity. The methods are now familiar<sup>1-3</sup>.

An alternative viewpoint on MHD stability is suggested by the calculation. All expectation values follow from the three invariants alone. In particular,  $\langle \tilde{v}^2 \rangle \neq 0$  even if we start with an initially quiescent state ( $\tilde{v} = 0$ ). We can seek values of the invariants, however, which minimize  $\langle \tilde{v}^2 \rangle$ , the expected mean square turbulent velocity field, for a given total energy. This implies a principle of extremal helicity. Making the magnetic helicity an extremum for given total energy provides guidance for designing MHD configurations for which  $\langle \tilde{v}^2 \rangle$  will remain as small as possible. The calculation is related to, but rather different from, one due to Taylor<sup>4</sup>. No decay of any invariant is invoked.

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# Nonlinear Behavior of Linearly Unstable Magnetohydrodynamic Modes<sup>†</sup>

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A nonlinear perturbational analysis is carried out in detail for the  $m = 1$  kink mode in a skin current cylindrical equilibrium. It is shown that for small growth rate  $\lambda$  the mode amplitude  $A(t)$  evolves according to

$$\ddot{A} = \lambda^2 A + \alpha A^3 \quad (1)$$

where  $\alpha = (2/\rho)(a^4 - 2)/(a^4 - 1)$  and  $a$  is the radius of the conducting boundary and the plasma radius is unity. The integrated form of (1) is  $\dot{A}^2 = C + \lambda^2 A^2 + (\alpha/2)A^4$  where  $C$  depends on the initial conditions  $A(0)$  and  $\dot{A}(0)$ .

For  $\alpha < 0$ , the amplitude  $A(t)$  undergoes periodic motion for  $C \neq 0$ , which can be seen simply from the curves around a figure-eight diagram in the phase diagram of (1). For  $C = 0$ ,  $A(t)$  initially grows exponentially, then pass a maximum and finally decay to zero exponentially.

For  $\alpha > 0$ ,  $A(t)$  approach infinity at some finite time, and this may be called an explosive instability. This shows that the mode grows with an accelerating speed. It also shows that in this case the mode cannot be stabilized at small amplitudes. If the motion will eventually stabilize, it must do so at a finite amplitude with the presence of all higher harmonics.

For  $\alpha < 0$ , the nonlinear stabilization comes in the form of periodic motion and it does not lead to a final stationary state. In fact, the motion cannot reach the bifurcation centers; nor does it pass through them. Because of this lack of dynamic connection between the motion and the bifurcation centers, the fluxes of these nonlinear motion are not the same as those for the bifurcated states; in fact the fluxes are proportional to the constant  $C$  which varies from curve to curve, showing the bifurcated states, the original equilibrium and the nonlinear motions have different fluxes.

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Nonlinear Behavior of Linearly Unstable MHD  $m = 2$  Mode<sup>+</sup>

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A weakly nonlinear analysis is carried out for the helical  $m = 2$  unstable MHD mode in a free boundary, constant current, cylindrical equilibrium such that, the singular surface is outside the plasma. Similarly to (1) it is shown that for small growth rate  $\lambda$ , the mode amplitude  $A(t)$  evolves according to

$$\ddot{A}(t) = \lambda^2 A(t) + \alpha A^3 \quad (*)$$

(The analysis of this equation is carried in (1).)  $\alpha$  is calculated in terms of the boundary of the outer cylinder. Though, technically, the number  $\alpha$  could be calculated through the standard bifurcating approach, and we compare it with such a calculation, there is a fundamental difference between the two; in bifurcation theory a pass from one state to another is tacitly assumed. But (\*) reveals immediately that these bifurcating states are not accessible dynamically. Indeed, in  $(\dot{A}, A)$  plane each trajectory corresponds to a different flux and two points can be connected only if they correspond to the same flux. But, points of bifurcation are centers - thus inaccessible.

It is therefore only through the present, global dynamics theory, that the meaning of bifurcation can be properly understood.

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(1) Y. Pao, this conference.

INVESTIGATION OF LINEAR AND NONLINEAR MHD  
INTERNAL INSTABILITIES\*

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The linear and nonlinear behavior of various MHD internal instability modes in two and three dimensions has been studied using a time-evolutionary, Eulerian computer code.<sup>1</sup> The code is based on ideal MHD but allows for resistivity, heat conduction and viscosity.

For the Rayleigh-Taylor mode with a smooth density variation linear growth rates have been determined and found to be in good agreement with the predictions of a quasiclassical approximation. The inclusion of artificial diffusion and damping terms permits the nonlinear evolution of the mode to be followed to its final stable state. A comparison is made between a flux-corrected-transport<sup>2</sup> treatment of the continuity equation and the use of artificial diffusion. The addition of shear to the magnetic field is observed to stabilize the mode and in some cases to lead to a displaced equilibrium.

In three dimensions the internal kink mode has been studied in both straight and toroidal geometry. For the former case a simple analytic equilibrium solution was used for the initial state, while in the latter case a numerical procedure for obtaining axisymmetric toroidal equilibria was devised. Linear growth rates have been determined for a variety of wavelengths,  $q$ -values, and boundary conditions. In addition, some results on the nonlinear evolution of the mode, including the possible generation of reverse axial magnetic fields, have been obtained.

\*Work supported by ERDA and NSF.

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## GENERALIZED GUIDING CENTER EQUILIBRIUM EQUATIONS

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## ABSTRACT

A selfconsistent set of equations is derived to describe an axisymmetric equilibrium plasma with strong variation of fields and plasma parameters normal to flux surfaces. A guiding center Hamiltonian  $H_0$  is obtained by a canonical transformation of the exact Hamiltonian to action-angle variables in the normal direction. An equilibrium drift-kinetic equation for the single-particle distribution function  $f$  of a species is written by equating the Poisson bracket  $[f, H_0]$  with a gyrophase averaged collision term, and appropriate moments of  $f$  give the species number density and current. Ampere's equation along with geometrical constraints on the metric tensor complete the set of equilibrium equations. The guiding center Hamiltonian is shown to give the correct drift velocity in the small gyroradius limit, and the lowest order finite gyroradius corrections are presented.

## MHD Equilibria of a Non-Circular Cross Section Stellarator\*

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Recent work has shown that closed vacuum magnetic surfaces exist in non-circular cross section stellarators.<sup>1,2</sup> It is the purpose of this paper to compute the MHD equilibrium of such configurations using the fully non-axisymmetric MHD code developed by Betancourt and Garabedian.<sup>3</sup> The code solves for the equilibrium of a perfectly conducting toroidal plasma with a diffuse pressure profile bounded by a fixed conducting wall. Equilibria are found by minimizing the energy utilizing the method of steepest descent as applied to a variational principle.

The last closed magnetic surface for an  $\ell = 2$  non-circular cross-section stellarator was used as the fixed outer wall in the code. This surface, subject to the constraint that the domain be star with respect to the minor axis, was approximated analytically. Other input parameters consist of the rotational transform and the pressure profile, which were also approximated analytically from previously calculated or experimentally observed parameters. The equilibrium was calculated for one toroidal field period, which excludes, for example, kink modes of wavelength greater than this distance.

An equilibrium was found for the case where no vacuum region was present. The equilibrium was seen to exist at beta values of up to 6%. Fourier decomposition of the plasma boundary was used to determine the relative growth or decay of various modes as a function of iteration. No major effect on the equilibrium was seen by varying the ellipticity of the boundary. When a vacuum region was introduced, local minima in poloidal field appeared as predicted by the calculation of the magnetic surfaces by field line following.

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\*Work supported by the National Science Foundation under grant ENG 75-11168 and ERDA under Contract Nos. E(11-1) 2387 and E(11-1) 3077.

## A Study of Mirror Machine Stability Problems\*

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A two component guiding center plasma with an electrostatic potential is used to represent the plasma in a mirror machine. A double asymptotic expansion is used to reduce the eigenvalue problem to a coupled set of ordinary differential equations. The first expansion is the "slow" approximation used in several high beta magnetohydrodynamic problems, equivalent to the "long, thin" approximation common in mirror work. The second expansion is equivalent to taking flux surfaces close to cylinders. These expansions have been used in many magnetohydrodynamic problems and have lately been applied by G. Vahala and M. Schmidt to mirror systems.

The stabilizing effect of strongly non-Maxwellian distribution functions is explored. Various spectral properties, points of accumulation, expansions for large azimuthal wave number are given. The connection with variational formulations is discussed. Implications for stabilization of axisymmetric mirror machines are given.

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# THE MHD EQUILIBRIA OF A SEPARATRIX LIMITED DOUBLET

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The MHD equilibria of a doublet shaped tokamak with the plasma boundary defined by an external separatrix has been obtained. There are two external separatrices with four stagnation points produced by four divertor-like coils. Therefore, four natural poloidal divertors are available for doublet and like PDX<sup>(1)</sup> they can be used as unload and screening divertors. The external coils are discrete and at sufficient distances away from the plasma surface such that ample shielding space is also available for coil protection. The total current of the two equatorial coils is only 40% of the plasma current which is modest in comparison with the 74% of the Doublet EPR design<sup>(2)</sup>. The major radius is 10 m for this model and was chosen arbitrarily for convenience. The MHD free boundary equilibrium configuration was obtained by solving the Shafranov-Grad equation using Green's function method throughout the entire space, inside as well as outside the plasma. The integral equation was solved iteratively using a Gaussian Quadrature technique. The results of this code have been compared with Princeton's MHD equilibrium code<sup>(3)</sup> on many "D" shaped tokamak designs<sup>(4)</sup>. They agree within numerical accuracy. In this calculation the poloidal  $\beta_p$  is 2, therefore like most finite  $\beta$  tokamaks, the magnetic axes (the peaks within the internal separatrix) are shifted outwardly in the horizontal direction from the geometric center. The internal separatrix and thus the peaks of the pressure and current density are near the outer edge of the plasma. There is a small amount of reverse current near the inner edge of the plasma. A three dimensional perspective view of the pressure profile is nearly flat in the vertical direction near the internal separatrix. The vertical separation of the peaks of current density are, however, more prominent.

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POSTER SESSION D

## APPLICATIONS OF THE KINETIC DESCRIPTION OF PONDERMOTIVE EFFECTS IN A PLASMA

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### ABSTRACT

The method of properly describing the interaction of electromagnetic fields with a plasma is receiving increasing interest because of the possibilities of using RF waves for tokamak heating, end-stoppering of mirrors and theta pinches, impurity control in tokamaks and tandem mirrors, and laser-fusion applications. While many theoretical analyses have used fluid equations for this description, in collisionless plasma regimes, the proper treatment of the dynamical equation for the pressure tensor including RF effects is extremely difficult. Also, the convention of using a kinetic equation, including only the pondermotive force has been shown to be inadequate; the full quasilinear equation must be solved, retaining the velocity space diffusion term.<sup>1</sup> In this presentation we show the solutions and describe the non-fluid, non-pondermotive force effects of the full kinetic description for special cases of non-resonant waves in weakly inhomogeneous collisionless plasmas. The situations of low thermal velocity plasmas, plasmas with inhomogeneous background confining magnetic fields, and short wavelength RF waves in mirror machines will be emphasized, and the importance of the new effects in understanding relevant experiments is shown to be significant.

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PSEUDOMAGNETIC FIELD AND A RELATION BETWEEN PONDEROMOTIVE  
FORCE AND LINEAR SUSCEPTIBILITY IN VLASOV PLASMA<sup>\*</sup>

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ABSTRACT

The canonical oscillation center formalism has been used to analyze many phenomena which occur in plasma physics. Using the Deprit<sup>1</sup> form of canonical perturbation theory, we investigate the equations of motion for an oscillation center in a slowly modulated wave. We find that the first correction to the oscillation center Hamiltonian,  $K_2$ , is related to the linear Vlasov susceptibility for each species by the formula:

$$4\pi \int d\Gamma f(\Gamma) K_2(\Gamma) = - \int d^3x \int d^3x' \underline{E}_\omega^*(\underline{x}) \cdot \underline{\chi}'_\omega(\underline{x}, \underline{x}') \cdot \underline{E}_\omega(\underline{x}') .$$

In this formula  $\Gamma$  denotes the phase space variables,  $f(\Gamma)$  is the quasi-static phase space density, and  $\underline{\chi}'$  is the hermitian part of the linear susceptibility tensor. This relation reduces the ponderomotive force problem to the linear susceptibility problem, which has been solved in many cases.

For particles with small velocity, we expand the above Hamiltonian. The zero-order (in velocity) term is the usual ponderomotive potential. The first order term yields a pseudo-magnetic field. We present explicit formulas for magnetized and unmagnetized plasma.

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<sup>\*</sup> This work is supported by the U.S. ERDA, Division of Magnetic Fusion Energy.

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## HYBRID SIMULATIONS OF QUASINEUTRAL PHENOMENA IN MAGNETIZED PLASMA\*

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We present a new class of numerical algorithms for computer simulation of low frequency ( $\omega \ll \omega_{ce}, \omega_{pe}$ ) electromagnetic and electrostatic phenomena in magnetized plasma. Maxwell's equations are solved in the limits of quasineutrality and negligible transverse displacement current (Darwin's model):  $\nabla \cdot \underline{J} = 0$ ,  $\nabla \times \underline{B} = 4\pi c^{-1} \underline{J}$ , and  $-\partial \underline{B} / \partial t = c \nabla \times \underline{E}$ . Electrons are modeled as a fluid with polarization effects ignored:  $0 \approx -n_e e \underline{E} + \underline{J}_e \times \underline{B} c^{-1} - m_e v_{ei} \underline{J}_e^{-1} - \nabla \cdot \underline{P}_e$ ,  $n_e \approx n_i$ . Ions are described as particles. A novel feature of these algorithms is the use of the electron fluid equation of motion to determine the electric field, which renders these numerical schemes remarkably simple and direct. The simulation plasma is either periodic, or bounded by particle reflecting conducting walls. Both fully nonlinear codes with spatial grids and linearized gridless codes have been implemented. Variations of the basic algorithm are presented which utilize the conservation of canonical momentum or which relax this constraint for more general situations. The numerical dispersion and stability of the various algorithms are investigated analytically and verified by computer experiments. Stability generally requires that  $(kV_A)_{\text{eff}} \Delta t \lesssim 1$  and  $\omega_{ci} \Delta t \lesssim 1$ , where the effective Alfvén frequency  $(kV_A)_{\text{eff}}$  includes space-time grid effects.

We have successfully simulated with linearized codes three examples of microinstability: two essentially electrostatic modes, the Dory-Guest-Harris instability and the drift-cyclotron-loss-cone mode; and the Alfvén-ion-cyclotron mode which is electromagnetic. The simulation results agree well with linear analytical theory, and the generally stabilizing influence of the electromagnetic modifications of the dominantly electrostatic modes is demonstrated. We have also applied these simulation models to the study of field-reversed magnetic-mirror systems. Our simulations verify that electron return currents cancel an embedded, linearly rising external current which is perpendicular to the vacuum magnetic field, only for times up to the Alfvén transit time of a plasma bounded by conducting walls.<sup>1</sup> After this, there is a growth of net current and concomitant magnetic field modification.

\*Work supported by ERDA under Contract No. W-7405-Eng-48.

<sup>1</sup>H. L. Berk and L. D. Pearlstein, Phys. Fluids 19, 1831 (1976).

SIMULATION OF LOWER-HYBRID HEATING OF A FINITE SIZE PLASMA\*

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A novel  $2\frac{1}{2}$  dimensional particle simulation code capable of handling a finite size magnetized plasma has been used to investigate a variety of important effects associated with the lower-hybrid heating scheme. The model is electrostatic and it permits the use of various external excitation geometries such as single charges, capacitor plates, and phased-array antennae. Some of the prominent effects isolated, of relevance to experiments planned for the near future, include the intense electron heating obtained by a suitable choice of the antenna parameters and the tendency for the heating to occur near the edge of the plasma. By taking advantage of the finite size of the plasma we have also investigated the heating obtained by coupling to the normal modes of the system. Various interesting nonlinear effects come into play in this situation; in particular, one detects nonlinear density changes associated with the ponderomotive force, as observed in recent experiments.<sup>1</sup>

\*Work supported by ERDA and ONR.

<sup>1</sup>R. L. Stenzel and W. Gekelman, Phys. Fluids 20, 108 (1977).

# VARIATIONAL METHOD FOR A CYLINDRICAL PLASMA\*

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## ABSTRACT

A variational procedure is presented which derives the plasma wave equations in a nonuniform cylindrical plasma. The present method is algebraically simpler than a direct calculation. The method is based upon the construction of a quadratic form in the field amplitudes representing the rate of energy production. The variation with respect to amplitude produces the correct Poisson integral equation in Fourier-Bessel space. The integral equation typically contains terms of the form

$$(1) \int dk_{\perp} k_{\perp} J_M(k_{\perp} a_s) J_{M+\ell}(k_{\perp} r) \phi(k_{\perp})$$

where  $a_s$  is the larmor radius of species  $s$ . By use of several Bessel function identities, we show that Eq. (1) may be approximated by a differential equation, to any desired order, in configuration space.

We apply the formalism to the study of flute interchange mode in a hot electron plasma. We consider two pressure profiles: one having a sharp gradient and the second having a slowly varying gradient. Approximate normal mode solutions are obtained when the mode is localized. The solution for the slowly varying gradient profile confirm the validity of the local approximation previously employed.

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ELECTROMAGNETIC ION BEAM INSTABILITY  
AND ENERGY LOSS OF FAST ALPHA PARTICLES\*

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In a linear theta pinch reactor, the 3.5 MeV alpha particles produced by D-T reactions have an energy loss range of several kilometers, and deposit their energy primarily in the electrons. If the alphas can drive microinstabilities, however, anomalous scattering may permit direct transfer of their energy to the ions, as well as provide a much shorter mean free path.

In a multiple mirror theta pinch, alphas with large perpendicular speeds are confined, whereas those with large parallel speeds escape in the axial directions, forming a beam. This drives an electromagnetic ion beam instability, which has been studied both linearly and via a second-order theory. If the beam is of sufficient density, and if the saturation level of the instability is sufficiently large, anomalous energy loss to the ions may take place over lengths much shorter than a classical mean free path.

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\*Work performed under the auspices of U.S.ERDA.

## THE EFFECTS OF TURBULENT HEAT FLOW ON IMPLOSION HEATING CALCULATIONS\*

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The cross field heat flow due to the lower-hybrid drift and the modified two-stream instabilities is incorporated into theta pinch calculations using the HYBRID code<sup>1</sup> with a self-consistent theory of turbulent heat flow recently developed at Science Applications, Inc. The turbulent heat flow has a smoothing effect during the implosion phase of a theta pinch: mainly causing a broader sheath structure of the magnetic field, density and temperature profiles. The simulation also shows that neutral deuterium will be more effectively ionized during the implosion with the turbulent heat flow than without the turbulent heat flow. A careful analysis of the density, temperature and magnetic field gradient shows that the modified two-stream instability will operate exclusively at the front of the piston where the density gradient is positive during the implosion, contrary to the previous notion of the dominance of the lower hybrid drift instability.

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\* Work supported by the U.S. Energy Research and Development Administration.

1. S. Hamasaki, N.A. Krall, C.E. Wagner and R.N. Byrne, Phys. of Fluids 20, 65, (1977).

Plasma Simulation on the CHI Computer.\* ROBERT W. HUFF, University of California at Los Angeles and The Rand Corporation. -- A  $2\frac{1}{2}$ -dimensional electrostatic particle simulation with fixed magnetic field plus mirroring is being programmed for the new Culler-Harrison Inc. (CHI) Plasma Simulation Computer. Completed microcoding of the critical portion promises times of 15  $\mu$ sec per particle-push. This is 4 times the speed of the corresponding IBM 360/91 assembly-language code, or better than a two-orders-of-magnitude decrease in cost. It will allow a significant million-particle run to be completed overnight. Other problems are also being programmed for the CHI computer, and promise similar time and cost reductions.

\* Supported by Energy Research and Development Administration.

Stochastic Ion Heating by Lower Hybrid Waves.\* by C. F. F. Karney and A. Bers,  
Plasma Fusion Center, Massachusetts Institute of Technology.

We have previously shown<sup>1</sup> that the motion of an ion in the presence of a constant magnetic field and a perpendicularly propagating electrostatic wave with frequency several times the ion cyclotron frequency is stochastic for fields satisfying  $E/B_0 > 1/4(\Omega/\omega)^{1/3}(\omega/k)$ . The stochastic region of velocity space extends from a lower limit given by  $\omega/k - (qE/mk)^{1/2}$  where the ion dynamics are determined by trapping to an upper limit which scales as  $E^{2/3}$ . For short times (of the order of 10 cyclotron periods) trapping is the dominant mechanism and this leads to a rapid increase in the energy of the tail particles. The stochasticity condition is independent of how close  $\omega$  is to a cyclotron harmonic, and unlike the work of Fukuyama et al.<sup>2</sup>, the results do not require that  $\omega$  be close to a cyclotron harmonic. The results are extended to the case of finite parallel propagation, where they are connected to those of Smith and Kaufman<sup>3</sup>, and to the case of an inhomogeneous magnetic field. The application to direct heating of the ions in a tokamak by lower hybrid waves is discussed.

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\*Work supported by U.S. Energy Research and Development Administration (Contract E(11-1)-3070)

1 C. F. F. Karney, A. Bers, and D. C. Watson, Bull. Am. Phys. Soc., 20, 1313 (1975); C. F. F. Karney and A. Bers, Bull. Am. Phys. Soc., 21, 1157 (1976) and Plasma Research Report 76/35-2 (Mar. 1977), Research Laboratory of Electronics, Massachusetts Institute of Technology.

2 A. Fukuyama, H. Momota, R. Itatani, and T. Takizuka, Phys. Rev. Lett. 38, 701 (1977).

3 G. R. Smith and A. N. Kaufman, Phys. Rev. Lett. 34, 1613 (1975).

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A Lagrangian type of MHD codes has been developed by treating elements of the fluid as particles.<sup>1</sup> The fully Lagrangian fluid is represented by a distribution of finite-size Gaussian-shaped particles which carry position, mass, momentum and energy (temperature) and introduce efficient coarse-graining of the particle quantities. A mesh is superimposed on the particles and fluid quantities are evaluated at mesh points by appropriate weighting from the particles. Drag forces between particles with different velocities in the same cell prevent extensive multi-streaming from developing; by smoothing the flow, it insures that particle quantities will not differ much from some average. A combination finite difference scheme, to calculate the magnetic terms, and fast Fourier transforms, to evaluate the particle pressure terms, guarantees momentum and magnetic flux conservation. The use of particles eliminates many difficulties often associated with Eulerian codes such as for example, negative densities. The method also means that any particle code of which there are many can be converted to a fluid code. The code has been extensively tested on problems of shock waves, and on propagation of sound waves and Alfvén waves. Applications of the code to hydrodynamics and magnetohydrodynamics in one, two and three dimensions, in cartesian as well as in toroidal geometry, are presented.

This work supported by ERDA and NSF.

<sup>1</sup>Codes of a somewhat similar nature have been investigated in connection with laser fusion studies by R. L. Morse et al., Proceedings of the Seventh Conference on Numerical Simulation of Plasmas, D 113, 114, 115 and 116, New York, 1975.

Theory of the Runaway Electron Tail\*, Kim Molvig, Miloslav S. Tekula, and Abraham Bers, Plasma Fusion Center, Massachusetts Institute of Technology.

A problem of considerable theoretical and practical significance is the structure of the high energy electron tail in a current carrying plasma. In the classical theory,<sup>[1]</sup> high energy electrons cannot be restrained by collision and runaway in the applied electric field, producing a tail extending to infinite momentum. While it is clear that noncollisional processes must account for the structure in the runaway region, a theoretical description has not previously been given. At the practical level, substantial energy losses can be caused by the runaways. This is particularly relevant to the anomalous energy transport associated with the electron component of a tokamak. For example, even in thermal equilibrium the energy flux from  $\omega = \omega_{pe} k_{||}/k$  plasma waves is larger than from electron thermal conduction.<sup>[2]</sup> The extended electron tail can destabilize these waves and thus enhance the energy loss.

This paper reports a self-consistent theory of the runaway tail together with the plasma wave spectrum it generates. Specifically, we treat the case of a radially finite magnetized plasma column in an applied electric field. Through the diffusion and friction forces caused by the waves, the particle distribution attains a steady state. Electron runaway to infinite momentum occurs only on the plasma column axis, while at other radii, the distribution is "cut off." Energy transport by the waves is larger than that by neoclassical ion thermal conduction and of the order of observed transport in intermediate density tokamaks ( $n \sim 3 - 10 \times 10^{13} \text{ cm}^{-3}$ ). In addition, the observed anomalous feature at  $\omega \approx \omega_{pe}$  in the radiation spectrum can be qualitatively explained. The momentum space structure of the steady state distribution function will be presented in a separate paper at this meeting.

\*Work supported by ERDA Contract E(11-1)-3070.

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2. M.N. Rosenbluth and C.S. Liu, Phys. Fluids 19, 815(1976).

ELEMENTARY DERIVATION OF THE KINETIC EQUATION  
FOR THE TWO DIMENSIONAL GUIDING CENTER PLASMA\*

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A kinetic equation for a two dimensional inviscid hydrodynamic fluid is derived in two ways. First the equations of motion for the modes of the fluid are interpreted as stochastic equations resembling the Langevin equation. To lowest order a Fokker-Planck equation can be derived which is the kinetic equation for one mode. Second, a suitable iteration scheme is applied to the Hopf equation which results in the same kinetic equation. A parameter describing the time scale is arbitrary and cannot be determined by the applied methods alone. It is shown that the kinetic equation satisfies the conservation requirements and relaxes to an equilibrium which is a rigorous solution of the Hopf equation.

\*Supported in part by a travel grant of NATO and the Minna James Heinemann Foundation and by ERDA contract No. AT(11-1)-2059-2.

Shear Effects on Low-Frequency Geometric Resonance Heating.\*

Reagan W. Moore, General Atomic Company--Results of geometric resonance heating at the cusp and Alfvén modes are presented. The guiding center equations for the plasma displacement are solved in slab geometry with magnetic field shear. The heating rate as a function of distance into the slab is calculated for several shear and density profiles. Conditions for which stimulated emission occurs are derived.

\*Work supported by the U. S. Energy Research and Development Administration, Contract EY-76-C-03-0167, Project Agreement No. 38.

Alfven Resonance Contribution to the Cavity Q  
in Low Frequency Magnetosonic Wave Heating\*

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The possibility of heating reactor-size tokamaks with magnetosonic cavity modes at frequencies below the ion cyclotron frequency has been discussed in Ref. 1. The dissipation of wave energy can be by two mechanisms (i) resonant wave-particle interaction, and (ii) excitation of the Alfven resonance<sup>2,3</sup> at the point in the plasma where  $\omega = k_{11} V_A$ . For a magnetosonic  $m = 0$  cavity mode the Alfven resonance and the turning point are close to each other<sup>1</sup>. Here we calculate the contribution to the cavity Q from the excitation of the Alfven resonance, and compare the result with the contribution from resonance wave-particle dissipation. The effect of finite ion Larmor radius<sup>4</sup> in relation to the closeness of the resonance and turning point is discussed.

\* This work was supported by E.R.D.A.

1. T.H. Stix, Plasma Heating in Toroidal Devices, Varenna, Italy (1976).
2. L. Chen and A. Hasegawa, Phys. Fluids 17, 1399 (1974).
3. J. Tataronis and W. Grossman, Nucl. Fusion 16, 667 (1976).
4. A. Hasegawa and L. Chen, Phys. Rev. Lett. 35, 370 (1975).

# Automated Calculation of Parametric Couplings

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A method for the automatic calculation of parametric instabilities has been devised and has been implemented using the MACSYMA system. A systematic survey of decay rates for modes propagating in warm fluid magneto-plasmas has begun and we present some results.

The calculations rely heavily on the structures of the linear modes involved in the non-linear interactions. Most of the information on these modes, e.g., frequencies, polarizations, currents is generated automatically and the algorithms developed are thus potentially useful in other calculations.

The method of computation including both analytic and programming aspects is given. For purposes of illustration we show several details of the method including typical input equations and portions of the resultant output.

This work was supported by ERDA Contract #E(11-1)-3004.

Two Dimensional Depletion of Lower Hybrid Pump by Quasi-Mode Excitations\*

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We study the effect of parametric excitation of quasimodes on the depletion of a lower hybrid pump wave driven from a finite extent source with parameters relevant to heating of tokamak plasmas. The set of coupled nonlinear equations<sup>1</sup> for the pump wave and the sideband mode reduces to those corresponding to the three wave resonant equations with one wave heavily damped. Exact analytic solutions<sup>2</sup> of these equations are investigated in detail to elucidate the effects of the pump width, the convective losses arising out of the finite group velocities of the modes and the noise level of the sideband. We present generalized "scaling" plots from which estimates of pump depletion can be obtained for various experiments.

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<sup>1</sup>L. Chen and R. L. Berger, PPPL-1306, Nov. 1976, Princeton U.

<sup>2</sup>F.Y.F. Chu and C.F.F. Karney, PRR 77/5, March 1977, Research Laboratory of Electronics, M.I.T.

\*Work supported in part by U. S. Energy Research and Development Administration (Contract E(11-1)-3070) and by National Science Foundation (Grant ENG75-06242-A01).

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Self modulation effects can become important for the propagation of lower hybrid waves in plasma, particularly for the high power levels envisioned in RF heating schemes. Earlier studies<sup>(1,2)</sup> have been done in two dimensions (in the plane defined by the electric field of the pump wave and the background magnetic field) and have led to nonlinear propagation equations, such as the MKdV or the nonlinear Schrödinger equation, which admit multiple-soliton solutions. These could physically manifest themselves by breaking up the resonance cones into filaments with intense localized electric fields and could further lead to localized heating. We have studied this problem in three dimensions with the motivation of including two additional physical factors. First of all, we are able to include the nonlinear effect arising out of the  $\underline{E} \times \underline{B}$  motion of electrons.<sup>(3)</sup> This leads to an enhancement in the threshold value for the formation of solitons. Secondly, we investigate the stability of the two-dimensional solitons to perturbations in the third dimension. We find that the third dimension introduces additional dispersive effects which render the solitons unstable to these perturbations.

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\*Work supported in part by U. S. Energy Research and Development Administration (Contract E(11-1)-3070) and by National Science Foundation (Grant ENG75-06242-A01).

<sup>1</sup>G. J. Morales and Y. C. Lee, Phys. Rev. Lett. 35, 930 (1975).

<sup>2</sup>H. H. Kuehl, Phys. Fluids 19, 1972 (1976).

<sup>3</sup>G. L. Johnston, C.F.F. Karney, F.Y.F. Chu, and A. Bers, Plasma Research Report PRR 76/8, Research Laboratory of Electronics, M.I.T. (1976).

## PLASMA ROTATION GENERATED DURING A THETA PINCH IMPLOSION\*

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Various mechanisms have been proposed to explain the origin of rotation in a theta pinch. Transfer of external angular momentum to the plasma, e.g. by the mechanism of end shorting, may in principle result in an approximately rigid rotation. However, such transfer is not necessary for the existence of rotation.

We show that a differential rotation of a plasma column, conserving total angular momentum, may result from the deviation of the ion distribution from a Maxwellian. Using a cylindrically and axially symmetric analytic model, we demonstrate that such a rotation is driven by the off-diagonal element of the pressure tensor  $p_{r\theta}$ . An experiment which has detected such rotation has been performed and has been analyzed with the Los Alamos one-dimensional Hybrid model. Since the transit time of the Alfvén wave from the ends to the probe is longer than the duration of the experiment, the ends may be ruled out as the source of the rotation. We present a microscopic description of the origin of the rotation in the experiment and its connection with magnetic field diffusion during the implosion, and we compare calculated rotational currents with the measured ones.

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\*Work performed under the auspices of U.S.ERDA.

Momentum Space Structure of the Steady State Runaway Distribution Function<sup>\*</sup>,  
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A steady state solution to the high energy electron tail was described in a paper by Molvig, Tekula & Bers (this conference). The steady state is achieved by the interaction of the electrons with the self-consistent turbulent wave spectrum. This spectrum is dominated by two regions of the magnetized,  $\omega = \omega_{pe} k_{\parallel}/k$  plasma wave spectrum. Modes with  $k_{\perp} > k_{\parallel}$  and hence  $\omega < \omega_{pe}$  will be referred to as " $\omega_{pe} \cos\theta$  modes"; the instability is locally convective. Modes which have  $k_{\perp} \approx 0$  and  $\omega \approx \omega_{pe}$  will be referred to as " $\omega_{pe}$  modes". These are absolutely unstable. Classically, electrons are freely accelerated beyond the runaway momentum,  $p_r$ . This results in a flat parallel distribution function,  $f_{\parallel}(p_{\parallel})$ , which can destabilize the  $\omega_{pe} \cos\theta$  modes by the wave particle interaction at the  $n = -1$  gyroresonance. These waves in turn generate an effective dynamic friction which eventually overcomes the acceleration by the electric field, leading to a cutoff in  $f_{\parallel}$ , at momentum  $p_c$ . Slowing down the electrons at  $p_c$  produces a pile-up and positive slope, which, in turn, destabilizes the  $\omega_{pe}$  modes. The  $\omega_{pe}$  modes evolve the distribution function to its marginally stable state between the  $p_r$  and  $p_c$ . Such a state can be achieved, for example, by collisional damping on the bulk electrons, resulting in a negligibly small slope and at reasonable level of turbulence. Since the dynamic friction is produced by convectively unstable  $\omega_{pe} \cos\theta$  modes, the cutoff momentum has a radial structure which leads to a channeling of the runaways. A detailed discussion of the physics and the momentum space structure of the steady state distribution function, which consists of vortices between  $p_r$  and  $p_c$ , will be presented.

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<sup>\*</sup>Work supported by U. S. Energy Research and Development Administration  
 (Contract E(11-1)-3070).

## OBLIQUE MAGNETO-ACOUSTIC HEATING OF A COLLISIONLESS PINCH\*

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There have been recent experimental observations of collisionless damping of Alfvén and  $m = 0$ ,  $k_z = 0$  magneto-acoustic waves at high beta.<sup>1</sup> In the absence of micro-instabilities and for non-zero  $k_z$ , we shall show that collisionless absorption can provide a heating mechanism when an equilibrium pinched plasma is pumped by oscillations of the confining magnetic field at a frequency near the oblique magneto-acoustic frequency.

In particular, we have analytically studied the dynamics of a sharp-boundary Vlasov-fluid screw pinch<sup>2</sup> using the following ordering scheme:

$$r_L/r_0 = \epsilon, \omega/\omega_c = 0(\epsilon), k_z r_0 = 0(1), \text{local beta} = 0(1), \vec{\nabla}_\perp = 0\left(\frac{1}{r_0}\right).$$

The equations so obtained are equivalent to those obtained by making the ansatz  $\gamma_{mh}(\omega) \rightarrow \gamma_M(\omega) = -2(\omega/k_z v_{th}) Z(\omega/k_z v_{th})$  in the sharp-boundary ideal magnetohydrodynamic equations.<sup>3</sup> The resulting physical model keeps ion kinetic effects along magnetic field lines while retaining only ion fluid effects transverse to the field lines.

To leading order all finite-Larmor-radius effects are absent. However, for non-zero  $k_z$  we find that the imaginary part of  $\gamma_M(\omega)$ , resulting from Landau damping, leads to collisionless absorption of energy by the ions when  $\omega/k_z >$  Alfvén velocity.

We shall present calculated values for the heating time scales and for the associated induced voltages.

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<sup>1</sup>W. Grossmann, M. Kaufmann, J. Neuhauser, Nuclear Fusion 13, 462 (1973).

<sup>2</sup>J. P. Freidberg, Physics of Fluids 15, 1102 (1972).

<sup>3</sup>Leaf Turner, Physics of Fluids 20, 654 and 672 (1977).

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PLENARY SESSION PB

T. DUPREE - STRONG TURBULENCE  
(NO ABSTRACT)

## On Field Reversal Via Neutral Injection\*

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A theoretical model of the buildup to field reversal by tangential neutral injection is presented. In the model, axial symmetry and  $z$  independence is assumed and the current is assumed to be carried by the ionic component. The injected neutral beam is represented by particles injected tangentially at fixed radius and fixed energy. The particles move subject to the constraint of constant canonical angular momentum and constant energy, i.e. the ion distribution function is represented by a product of delta functions in energy and canonical angular momentum. Extending a result given by Marx<sup>(1)</sup>, it is shown that within the region traversed by the ions the ion current is everywhere proportional to  $(v_\theta/r)$ . From this result and from the conservation equations an integral equation is derived which determines the particle trajectories everywhere within the current layer. This equation is solved analytically and used to predict certain properties of the layer. It can be shown that: a) Field reversal is not possible in this model unless the outer radius,  $a$ , of the layer is greater than  $2 \cdot a_0$ , where  $a_0$  is the gyroradius of the ion in the (uniform) vacuum field;  $a/a_0$  equal to at least 2.5 is required for a reasonable degree of field reversal. b) When  $a/a_0 \gg 2$ , high degrees of field reversal, in excess of 2:1, are achievable. It is shown that the above theory can be extended to the case of a vacuum field which varies with radius or to the case of multi-beam injection, where each ion component moves in the combined field of other ions and the imposed vacuum field.

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<sup>(1)</sup> K.D. Marx, Phys. Fluids 11, 357 (1969)

\*Work performed under the auspices of the U.S. Energy Research and Development Administration under Contract No. W-7405-Eng-48.

Report on the Lower Hybrid Heating Theory Workshop\*

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We present a summary and review of the important theoretical results and unsolved problems that were discussed recently at a special workshop on lower hybrid heating for tokamaks. This will cover: (a) linear coupling, propagation, and damping; (b) nonlinear effects in propagation, such as self-modulation and parametric interactions; and (c) heating mechanisms, such as stochastic heating of electrons and ions directly by the lower hybrid waves, and indirectly by the parametrically excited waves.

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# Nonlinear Saturation of the Dissipative Trapped-Electron Instability\*

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Nonlinear saturation of short-wavelength dispersive  $[k_{\perp}^2 \rho_i^2 (1 + T_e/T_i) \sim 1]$  trapped-electron modes in tokamak is considered including full two-dimensional nonlinearities and nonresonant mode-coupling interactions. Nonlinearities due to both ions and trapped electrons are shown to be important. We find two effective, but competing saturation mechanisms; namely, trapped-electron induced scattering and nonlinear ion Landau damping. The former mechanism, which is the novel feature in the present analysis, scatters wave-energy into shorter wavelength modes, while the latter mechanism is known to scatter wave-energy into longer wavelength modes. Both of the mechanisms involve intrinsically two-dimensional nonlinear coupling processes. For typical tokamak parameters, the trapped-electron induced scattering is shown to dominate, and the unstable wave energy is transferred to the shorter wavelength modes, where it can be effectively dissipated by ion viscosity.

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## NONLINEAR DYNAMICS OF RUNAWAY ELECTRONS IN TOKAMAKS

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We present a linear and nonlinear analysis of the dynamics of the runaway electrons and a theoretical interpretation of many anomalous phenomena observed in low-density Tokamaks: reduced resistivity, enhanced synchrotron radiation, intense burst of rf and x-ray emission, ion heating and the hole burning in the liners of TFR and Alcator. A definite runaway distribution function is obtained by solving the Fokker-Planck equation as the unperturbed distribution for the stability analysis and nonlinear evolution. The stability boundary thus obtained is in good agreement with experiments. In the stable region, the classical resistivity is found to be reduced from the Spitzer value. In the unstable region, quasilinear equations are solved in a simplified form. The turbulent pitch-angle scattering of the runaways increases their perpendicular energy and their synchrotron radiation. The time-dependent radiation is calculated and the level is found to be enhanced by one to two orders of magnitude above thermal in  $\mu\text{sec}$  with a nonthermal spectrum, in qualitative agreement with experiments. Later on, the quasilinear reduction of the parallel energy for  $v_{\parallel} > (\Omega/k_{\parallel})_{\min} \sim (\Omega/\omega_p)(E_0/E)^{1/2} v_{\text{th}}$  leads to a nonlinear instability due to the formation of a bump in tail. This instability can drag electrons with large  $v_{\perp}$  to small parallel velocity  $\sim \sqrt{E_0/E} v_e$ , which can then be trapped by the toroidal ripple field if  $\delta B/B > 1/4\pi(\omega_p/\Omega_c)^2$ . The vertical drift of these trapped energetic electrons can lead to the hole burning if a sufficient heat flux is available. We calculate the heat flux from the quasilinear final-state distribution and found it to be sufficient to burn the liner for TFR parameters.

## HEATING TOKAMAKS VIA THE ION-CYCLOTRON AND ION-ION HYBRID RESONANCES

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The ion-ion hybrid resonance can occur at high plasma densities in tokamaks and thereby absorb energy from the fast magnetosonic mode which would otherwise propagate freely. Ion-cyclotron resonance regions, although they occur in the low density peripheral plasma, can nonetheless cause sufficient absorption to compete with fundamental cyclotron resonance damping by ions. For the ion-ion hybrid resonance it is shown that: (1) The energy absorption occurs via a sequence of mode conversions. (2) A poloidal field component normal to the ion-ion hybrid mode conversion surface strongly influences the mode conversion process so that roughly equal electron and ion heating occurs in the present proton-deuterium experiments, while solely electron heating is predicted to prevail in deuterium-tritium reactors. (3) The ion-ion hybrid resonance suppresses toroidal eigenmodes. (4) Wave absorption in minority fundamental ion-cyclotron heating experiments will be dominated by ion-ion hybrid mode conversion absorption for minority concentrations exceeding roughly 1%.

For the ion-cyclotron resonance, it is shown that: (1) Ion-cyclotron mode conversion leads to surface electron heating. (2) Ion-cyclotron mode conversion absorption dominates fundamental ion-cyclotron absorption thereby preventing efficient ion heating.

As a specific example, in "second harmonic heating experiments" of deuterium plasmas containing small concentrations of protons, wave absorption proceeds via the ion-ion hybrid mode conversion and the actual ion heating results from cyclotron damping of the converted wave. True second harmonic heating in the PLT tokamak can be obtained by using a proton-He<sup>3</sup> plasma which has the same ratio of gyro-frequencies as a deuterium-tritium plasma and should accurately simulate reactor ion-cyclotron heating physics.

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\*\*Sponsored by the National Science Foundation.

## MAGNETO-ACOUSTIC HEATING OF HIGH BETA PINCHES \*

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We have studied the heating of a cylindrical plasma column by  $m = 0$ ,  $k_z = 0$  oscillations of the confining magnetic field at a frequency near the magneto-acoustic frequency. Two mechanisms have been studied, collisional and collisionless, for heating both diffuse and sharp-boundary plasmas.

The heating times and associated R.F. voltages will be presented for two dissipation mechanisms: a collisional viscosity, and a collisionless absorption induced by the energy-anisotropy-driven electromagnetic ion cyclotron instability.<sup>1</sup> Our results in the collisional case agree well with experiment.<sup>2</sup> In the collisionless case, there is at least qualitative agreement with experiment.<sup>3</sup>

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<sup>1</sup>R. C. Davidson and J. M. Ogden, Phys. Fluids 18, 1045 (1975).

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<sup>3</sup>W. Grossmann, M. Kaufmann and J. Neuhauser, Nucl. Fusion 13, 462 (1973).

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THEORY OF RF BURSTING IN MIRROR PLASMAS<sup>+</sup>

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ABSTRACT

Mirror plasmas subject to neutral beam heating are known to exhibit periodic bursts of RF activities at frequencies near the ion cyclotron frequency. We investigate the possibility<sup>1</sup> that this phenomenon is due to an instability caused by the nonlinear coupling of the drift cyclotron loss cone mode (DCLC), with frequency near the ion cyclotron frequency, and a very low frequency Rayleigh-Taylor (RT) mode. This coupling, which is a form of nonlinear resonant cyclotron damping, is investigated under four conditions: (a) the DCLC is a negative energy mode; (b) the DCLC is a zero energy mode; (c) the DCLC is a negative energy mode, and the sum of the DCLC frequency and the RT frequency is exactly the ion cyclotron frequency; (d) the DCLC is a zero energy mode, and the sum of the DCLC frequency and the RT frequency is exactly the ion cyclotron frequency. In the first case, a periodic instability is found;<sup>1</sup> in the last three cases, we find explosive instability for most combinations of physical parameters.

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+ Work supported by U. S. Energy Research and Development Administration under Contract No. AT-(04-3)-1018.

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1. C. S. Liu and R. E. Aamodt, Bull. Am. Phys. Soc. 21, 1099 (1976).

# Formation of Convective Cells and Anomalous Diffusion Due to Drift Turbulence\*

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Drift wave turbulence has been of current interest in toroidal confinement devices because of the strong correlation between the observed density fluctuations and anomalous plasma transport. In order to understand the anomalous diffusion process in detail, large scale, fully three-dimensional particle simulations<sup>1</sup> has been carried out for the collisionless drift instabilities in a cylindrical plasma with and without sheared magnetic fields.

For the shearless and weak shear cases ( $L_s/L_n \gtrsim 25$ ), we have found that the nonlinear excitation of convective cells ( $\omega = 0$ ) due to drift instabilities gives rise to strong plasma turbulence and anomalous diffusion. Based on the simulation results, a theory<sup>2</sup> has been developed to show that nonlinear coupling of drift instabilities can resonantly excite the convective cells at the wavelength of the order of  $k_\perp \rho_i \approx 1$  where drift instabilities have the maximum growth rate. The observed diffusion is mainly due to drift instabilities at an earlier stage and is due to the nonlinearly excited convective cells at the later stage when the plasma becomes turbulent. The observed overall diffusion coefficient is in good agreement with the convective cell diffusion formula when the enhanced level of fluctuation are used.

For the strongly sheared cases [ $25 > L_s/L_n > (m_i/m_e)^{1/3}$ ], drift instabilities cause plasma diffusion which leads the plasma to a marginally stable state where  $L_s/L_n \approx (m_i/m_e)^{1/3}$  and drift instabilities is stabilized by shear. Also, no convective cell has been observed.

The observed broad power spectra in these simulations have resemblance with those measured recently in toroidal devices. Recent experiments in FM-1 indicate that for very strong shear, the power spectrum has a well-defined peak near  $\omega^*$ . As shear is reduced, a new peak near  $\omega \approx 0$  appears and further reduction of shear results in a broad spectrum with no well-defined peaks. Similar results are reported from the ATC experiments.

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\* Work supported by U. S. ERDA Contract No. E(11-1)-3073.

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2. H. Okuda and C. Z. Cheng, PPPL-1328 (1977).

POSTER SESSION E

NUMERICAL CALCULATION OF OXYGEN AND IRON  
IMPURITY DIFFUSION IN TOKAMAKS\*T. Amano<sup>†</sup> and E. C. CrumeOak Ridge National Laboratory  
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To model multispecies impurity transport in tokamaks, a set of diffusion equations including source and sink terms from atomic processes (rate terms) are solved numerically. The diffusion terms and the rate terms are integrated separately in time using a fractional step-splitting technique which assures second order accuracy in the time step  $\Delta t$ . The rate terms are integrated using an eigenvalue method<sup>1</sup> which allows such large time steps that the diffusion time scale constrains the step size. Rate coefficients were taken from Hinnov<sup>2</sup> and the diffusion coefficients are those of Hirshman<sup>3</sup>.

Calculations have been performed using all the ionization states of oxygen and iron as impurities in a hydrogen plasma. We find that the lower ionization stages of the impurities in the colder edge region rapidly diffuse inward and are further ionized in the hotter regions of the plasma. However, the diffusion of the more highly ionized states in the hotter regions proceeds only slowly, and we do not find appreciable accumulations in the plasma center even after 40 msec for typical ORMAK parameters. The enhancement of the diffusion of the iron impurities due to collisions between the iron and oxygen ions is also apparent.

The effects on impurity diffusion of anomalous electron diffusion are now being investigated.

\*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

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<sup>1</sup>M. Okamoto and T. Amano, Japan Atomic Energy Research Institute JAERI-Memo 6828 (1976).

<sup>2</sup>E. Hinnov, Princeton Plasma Physics Laboratory Report MATT-777 (1970).

<sup>3</sup>S. P. Hirshman, Princeton Plasma Physics Laboratory Report PPPL-1291 (1976).

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The prediction<sup>1</sup> that saw-tooth oscillations should be quenched as the density is raised, while the electron temperature does not fall off considerably, is based on two theoretical observations: a) the growth rate of the  $m^0 = 1$  resistive modes is considerably reduced by the combined effects of finite ion gyroradius, electron-ion drift wave frequency and ion-ion collisions; and b) the "growth rate" of the thermal instability, that precedes the observed  $m^0 = 1$  oscillation and produces the rising phase of the saw-tooth, decreases as density increases.<sup>2</sup> Recent experiments in Alcator tend to confirm the above prediction and reinforce the suggestion that, in view of the improvement of the confinement time with density, the role of resistive modes in it should be considered in addition to other possible factors.<sup>1</sup>

We have studied both numerically and, in appropriate limits, analytically the stability and radial profiles of the  $m^0=1$  internal kink mode ( $\hat{\lambda}_H \geq 0$ ) and of the resistive "reconnecting" mode ( $\hat{\lambda}_H < -1$ ) in high density regimes, where  $\hat{\lambda}_H \equiv \lambda_H/\epsilon^{1/3}$ ,  $\lambda_H \equiv \gamma_{MHD}\tau_H$ ,  $\epsilon \equiv \tau_H/\tau_R$ ,  $\gamma_{MHD}$  is the ideal MHD growth rate which tends to be negative in toroidal configurations,  $\tau_H$  is a typical Alfvén time, and  $\tau_R$  is the resistive diffusion time. While the relevant growth rates are sharply reduced by the "kinetic" effects mentioned in a), these modes acquire a frequency of oscillation that is expressed by  $\omega = \omega_{*e} F(\hat{\lambda}_H, \hat{D})$ , where  $\omega_{*e}$  is the electron drift wave frequency,  $F$  is a function of  $\hat{\lambda}_H$  and  $\hat{D} \equiv D/\epsilon$ ,  $D$  being the normalized collisional viscosity parameter. In particular,  $F \approx 1$  for  $\hat{\lambda}_H \ll -1$  and  $F \approx \hat{D}$  for  $\hat{\lambda}_H = 0$  and  $\hat{D} < 1$ .

\*Work supported in part by E.R.D.A.

<sup>1</sup>B. Coppi, Lecture at the Varenna Symposium on Plasma Heating, Sept., 1976

<sup>2</sup>O. Manley, private communication at the Varenna Symposium on Plasma Heating, Sept., 1976.

# Effect of Field Errors on Divertor Scrape-off Width\*

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Away from the x point of a poloidal divertor, plasma diffusion, whether classical or microturbulent, gives a scrape-off layer thickness of order  $10^{-2}$  of the plasma radius. Magnetic perturbations produced by the field coils or by MHD activity in the plasma can broaden the scrape-off layer by creating a region of ergodic field lines about the separatrix. The width of this ergodic layer, in which the field lines cover a finite volume instead of just a surface, is essentially identical to the region in which the magnetic islands produced by the perturbation overlap. We have developed a simple but general method of finding the region of island overlap by solving a magnetic differential equation using Hamada coordinates (see PPL-1343). In a simple analytic divertor model, we have found that away from the x point the ratio of the width of the scrape-off layer due to ergodicity to the plasma radius is roughly

$$0.1 n q_0 \frac{B_{\perp}}{B_p} \exp \left[ - \frac{\pi}{2} \left( \frac{n q_0}{4} - \frac{m}{2\sqrt{2}} \right) \right]$$

with m and n the poloidal and toroidal perturbation mode numbers and  $B_{\perp}$  is the component of the perturbation orthogonal to the original magnetic surfaces.  $B_p$  and  $q_0$  are the values the poloidal field and the safety factor would have, at the location of the x point, if the divertor coil were turned off.

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# Diffusive Tokamak Equilibria, Generalized Bootstrap Currents and the Relation between Particle Diffusion and Magnetic Field Diffusion

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## Abstract

A condition for general axisymmetric diffusive equilibria which relates the outward diffusion flux with the toroidal current density is derived. In an approximate version it requires that some effective diffusion velocity  $V_D^*$  must not exceed the poloidal magnetic diffusion velocity  $V_m$ . Relevant consequences follow in the regime of anomalous diffusion if diffusion is caused by an anomalous parallel electron viscosity instead of an anomalous perpendicular resistivity. In the former case  $V_D^*$  equals the real diffusion velocity  $V_D$  and an anomalous bootstrap current arises which leads to rather low upper limits for  $\beta_p$ . If the usual trapped ion or Bohm diffusion is assumed to be caused by an enhanced viscosity, no stationary equilibria would be possible in a system governed by the respective diffusion law.

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# Nonlinear Dynamics of Impurities in a Tokamak :

by N. Byrne, S.I.A. , San Diego and M. Cotsaftis, Fontenay-aux-Roses, France

Based on the drift kinetic equations, several approximations on the collision operator corresponding to the various neoclassical regimes for the different species in a Tokamak plasma have been considered, and the resulting systems of transport equations have been both theoretically and numerically investigated . But there still exists a strong discrepancy between the resulting fluxes and densities and the experimental careful and numerous observations . To tentatively reduce this gap, it is proposed in this communication to take advantage of the following remark . As shown elsewhere <sup>/1/</sup>, for high enough temperature (  $\approx 1.4$  KeV in TFR) electrons are no longer neoclassical, but fall into a high loss regime ruled out by the Trapped Electron Instability (T.E.I.) . Also, the analysis of the plasma ions shows that they are going into a modified regime by the effect of T.E.I. <sup>/2/</sup> . This suggests that the impurity ions could as well be in a modified regime . Effectively, for :  $\alpha = \frac{n_e z_i^2}{n_i z_i^2} \approx 0.5 - 1$ , corresponding to the present experimental value, the theoretical (neoclassical) value of the electroflux is :  $\Gamma_e^{N.C.} \approx \sqrt{\frac{m_e}{m_i}} \Gamma_i z_i \ll \Gamma_i$  , and thus does not enter the total flux balance, which is realized in this case through the ions only, in disagreement with estimates from observations . But in the T.E.I. (experimental) regime, the flux now becomes  $10^2$  larger at least, and can no longer be neglected in the balance as previously . So the effect of T.E.I. on the impurity fluxes has been evaluated for different approximations of the collision operator, and taking advantage of the fact that the impurities in actual experiments are in collisional regime at the periphery of the plasma where they are created . Also, the amplitude of the fluctuating electric field has been taken as the one corresponding to the nonlinear limit of the diffusion coefficient  $D \approx \frac{\gamma}{k^2}$  , which is in relatively good agreement with observations . A new system of transport equations for the impurity ions has been derived, which now contains a new term proportional to the impurity density gradient . Theoretical analysis of this system shows, as experimentally observed, the correct ordering of the impurity density peaks, and, due to the new term, the enhancement of the fluxes in the gradient zone, pushing the peaks out of this region . This could provide an interesting test for justification of the T.E.I. regime . Numerical investigations are in progress for more detailed analysis .

/1/ M. Cotsaftis : in Plasma Physics, Nonlinear Theory and Experiments, edited by H. Wilhelmsson, Plenum Press New-york (1977), p.326

/2/ N.A. Krall , J.B. McBride : Report LAPS-22 , SAI-76-867-LJ, December 1976

DIFFUSION OF MAGNETIC FIELD LINES  
IN A CYLINDRICAL PLASMA CARRYING CURRENT\*

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A model for studying magnetic field line diffusion in a low  $\beta$  cylindrical plasma has been constructed. The plasma contains a strong axial field and carries current primarily in the axial direction. The field and current are solved for self consistency. The model which emerges under certain approximations is analogous to ideal two dimensional hydrodynamics, where the magnetic field lines play the role of the paths of vortex particles in space-time, current density plays the role of vorticity density, vector potential plays the role of velocity potential and distance along the cylinder becomes time. A computer simulation is performed which can follow thousands of field lines (vortex particles) down the tube. Thus one can gain information about the destruction of magnetic surfaces and the subsequent ergodization of the magnetic field structure in the presence of several localized self consistent current perturbations.

\*Work supported by ERDA.

# Ion-Mixing Mode and Model for Density

## Rise in Confined Plasmas\*

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A model to explain the rate of plasma density rise that is observed in experiments, such as Alcator, when neutral gas is fed into the plasma chamber, is presented for regimes with high densities where known collisional transport processes do not provide an adequate explanation. We assume that in the presence of a vigorous heating power input, such as the Ohmic heating is in Alcator, a dense layer of cold plasma is produced at the edge of the plasma column. This has the effect to produce a relatively sharp ion temperature gradient and/or a "shoulder" in the local density profile. Then fluid-like ion drift modes are excited and lead to mixing of the "hot" and "cold" ion populations while a net inflow of electrons and ions is produced by the effects of finite longitudinal electron thermal conductivity.

Assuming a saturation amplitude for the modes which orders the microturbulent  $\bar{E} \times \bar{B}$  drift velocity to be comparable to the diamagnetic drift velocity, we show that the resulting inward particle diffusion coefficient is in the range  $(\frac{m_e}{m_i})^{1/2} D_{BOHM} \lesssim D < D_{BOHM}$  where the local values of the electron temperature is adopted. This yields the needed inward transport of particles on the rise-time of the center density. The relevant anomalous particle fluxes have been incorporated in a numerical transport code and preliminary results are presented.

\* Work supported in part by E.R.D.A.

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Nonlinear Transport in Tokamak Systems : M. Cotsaftis, Asso. EUR-CEA, Fontenay-aux-Roses, France, N.A. Krall, J. McBride, SAI, San Diego, USA .

Theoretical analysis of the complete MHD system shows that to first order, it splits into three parts: the mass-energy, the dynamic, and the impurity parts . Then its dynamics is ruled out by the first part, with branchings determined from the second part, whereas the last part can be studied aside . In the course of a discharge, the first branching occurs when current-thermal instabilities are strong enough to destroy skin effect . Then the core of the plasma is isolated from the exterior by an island zone at the periphery, located for  $q(a) \approx 3-4$  between  $r(q=2)$  and  $a$  . The electronic temperature then grows up almost freely, following a nice transformation group during its evolution until the gradient  $\frac{dT_e}{dr}$  becomes large enough to allow a new branching toward a high loss diffusion regime ruled out by Trapped Electron type Instability (T.E.I.), as verified<sup>/1/</sup> . Moreover, at the same time, the safety factor  $q$  becomes  $< 1$  in the center of the plasma, which is now divided into three zones : the outer (islands) zone, the gradient zone between  $r(q=2)$  and  $r(q=1)$ , and the central zone where  $q < 1$  . In neither of these three zones the electronic thermal conductivity coefficient  $\chi_e$  is classical (or neoclassical) . The new value of  $\chi_e$  is evaluated in each of the three subdomains : in the outer zone under the current-thermal instability, in the gradient domain with the T.E.I. regime and in the central zone where internal MHD  $m = 0$  mode is unstable (for  $q < 1$ ) and couples to a (drift) resistive  $m = 1$  mode located around  $r(q=1)$  to produce as observed the sawtooth relaxations on the electronic temperature . The corresponding  $\chi_e^{Tot}$ , which is the sum of the  $\chi_{e,j}^{Exp}$  ( $j=1,2,3$ ) in each of the three previous subdomains, is compared with a global  $\chi_e$  derived from the analysis of the energy balance of TFR discharges . A compatible agreement is found, validating the possibility of this multilayers structure of the discharges, and each layer with its own properties because they are well separated . Moreover, it is shown that the electron anomaly creates in turn an anomaly on the plasma ions (and impurities) which are then following a modified neoclassical regime . The analytical expression for  $\chi_i$  is given<sup>/2/</sup>, and compared with experiments . Some consequences in ions heating are discussed :  $\tau_{Ei}$  does not vary with  $T_i$  as expected and  $P_{ei}$  increases, as exhibited by the observations . Finally, the contribution to the impurity fluxes coming from the electron anomaly is given .

/1/ M. Cotsaftis : Plasma Physics, Nonlinear Theory and Experiments (Nobel Symposium) H. Wilhelmsson, editor, Plenum Press, New-York (1977) p.326

/2/ N.A. Krall, J. McBride : Report LAPS-22; SAI-76-867-LJ (1976)

## Neoclassical Theory of Equilibrium and Transport in a Z-pinch Plasma\*

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## Abstract

The neoclassical theory of tokamak transport is adapted to study field-geometry effects on equilibrium and transport in a Z-pinch. Since, typically, the poloidal and toroidal magnetic field components are comparable, radial diffusion and heat conduction in a large aspect-ratio Z-pinch results mainly from classical, rather than neoclassical, transport. Neoclassical effects remain of interest, however, especially with regard to ion-electron energy exchange. The study is based on a new version of the drift-kinetic equation, which is derived using orderings appropriate to the ZT-1 device, and which allows for rapid toroidal motion of the plasma. In lowest order, the equation describes an equilibrium configuration which generalizes the standard Shafranov geometry, and requires poloidal variation of the density and electrostatic potential. These variations, together with the variation of the magnetic field, are taken into account in solution of the higher order drift-kinetic equation, from which neoclassical transport effects are obtained.

\*This work supported by Los Alamos Scientific Laboratory.

INJECTION, COMPRESSION AND TRANSPORT STUDIES\*

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There are several areas of current interest which call for an improved treatment of the magnetic geometry in transport calculations. High- $\beta$  equilibria found by Clarke and Sigmar [1] produce a significant change in the toroidal position of the plasma. The calculation of major radius compression requires an axi-symmetric framework, as does the comparative study of various non-circular cross-sections. The Oak Ridge Tokamak Transport Code now calculates axi-symmetric transport, using a version of the ISLAND code developed at the Courant Institute. The code also incorporates the Fast Ion Fokker-Planck Code, developed by R. H. Fowler and J. Smith of ORNL, so that injection and compression can be treated self-consistently for the first time.

RESULTS

1) The  $q$ -conserving equilibria of Clarke and Sigmar can be reproduced in a transport calculation. However, impurity beam-trapping and transport scaling will play an important role in determining the limiting  $\beta$ . We will discuss recent theoretical and experimental results concerning the trapping cross-sections used in these calculations.

2) The calculation of injection/compression has been tested against ATC experimental results.<sup>3</sup> The role of heavy metal impurities in explaining electron heating anomalies in ORMAK and ATC is suggested by comparing these calculations with the experimental results. Relaxing the usual model of instantaneous adiabatic compression allows improved estimates of the expected neutron production from D-T tokamaks.

3) A number of comparative transport studies have been done: PLT results have been compared with the neoclassical transport theory [2]; Doublet IIA shape comparisons and stellerator/tokamak comparisons have been calculated with the pseudo-classical transport model.

1) J. F. Clarke, D. J. Sigmar, Phys. Rev. Lett. 38 70 1977.

2) F. L. Hinton, R. Hazetine, Rev. Mod. Phys. 48 239 1975.

3) L. R. Grisham, K. Bol, J. Hogan, PPL-1330, March, 1977.

\*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

## IN THE MATTER OF THE MISSING NEUTRONS FROM ALCATOR AND TFR

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The neutron flux emitted from dense deuterium plasmas in the Alcator device is many times smaller than the value expected from the charge-exchange  $T_i$  as well as from simple equilibration considerations.<sup>1</sup> A large neutron "shortfall" is also found in deuterium plasmas heated by intense hydrogen-beam injection in the TFR device<sup>2</sup> ( $T_i > 1$  keV). We have resolved part of this mystery by considering the effect of the large toroidal-field ripple in these devices on the tail of the bulk-ion population.

The TF ripple well depth  $\delta$  is 0.04 in the central region of Alcator, whilst in TFR,  $\delta \approx 0.005$  near  $r = 5$  cm, and increases rapidly with  $r$ . Hence sufficiently energetic members of the bulk-ion population can diffuse rapidly away from the hot central region by the superbanana process ( $D_{\perp} \propto E^{7/2}$ ). We show that this process results in truncation of the ion velocity distribution in a range important to fusion production, and thus leads to a marked reduction in the fusion reactivity. This ripple-induced tail-depletion effect accounts for a large fraction of the neutron deficits in the Alcator and beam-heated TFR plasmas. This phenomenon is also expected to result in significantly reduced D-T alpha-particle production in compact devices with close-fitting TF coils.

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<sup>1</sup>Alcator Group, to be published.

<sup>2</sup>TFR Group, in Theor. and Expt. Aspects of Heating of Toroidal Plasmas (Grenoble, 1976) 2, 141.

# Transport During Turbulent Heating in a Tokamak\*

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## ABSTRACT

A model for the Texas turbulent torus is described. The prediction of the model shows a strong skin effect, and is in agreement with the experimental data if one uses the electron trapping condition to saturate the Buneman instability and the anomalous collision frequency  $\nu^A \simeq 10^{-5} (T_e/T_i) (u/v_e) \omega_{pe}$ , which correlates with ion tail formation for the saturated state of the ion sound instability.

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## Fusion Product Transport and Slowing in Fusion Plasmas\*

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Understanding the transport and thermalization of fusion products (fp's) in fusion devices is a central problem in the search for ignited operation and also in the evaluation of other effects such as the buildup of "ash," bombardment and erosion of the first wall, and induced plasma rotation. Our studies of these effects have focused on the development of techniques to treat three distinct problems for the specific case of Tokamak geometry: a) rapid computation of fp-orbits during thermalization, b) methods to evaluate the angle and spatial distributions of escaping fp's, and c) evaluation of potential microinstabilities during slowing down.

An accurate thermalization model is basic to all of these calculations. We employ a simulation technique, but to circumvent the problem of following many orbits, a phase-group trajectory method is used along with a weighted energy-loss model. Ripple trapping is incorporated by use of a probability density to describe trapped-particle residence times. Calculations for representative TFTR- and reactor-type plasmas show that both the  $\alpha$  heating and thermalization profiles (along the minor radius) are considerably flatter than predicted by the common "in situ" approximation. Distribution functions for escaping alphas are calculated from the same model using a systematic classification of orbit classes to identify loss cone boundaries. The escaping fp's generally display a peaked profile in the poloidal direction superimposed on a periodic ripple-like profile in the toroidal direction. The spatial dependence of fp losses is combined with an estimate of the perpendicular conductivity in order to evaluate the radial electric field induced by the losses. The resultant plasma rotation is damped by several forces including ion-neutral collision (near the edge), charge exchange (edge also), and angular momentum diffusion (dominates near the plasma center). Toroidal rotation speeds approaching thermal speeds are predicted for reactor-like plasmas, possibly representing a useful diagnostic.

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\* Work supported under the auspices of US ERDA.

# Impurity Radiation Effects in Tokamaks<sup>\*</sup>

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Impurity radiation loss rates for typical tokamak parameters have been numerically calculated using an "average-ion" model. Coronal equilibrium emission rates for impurities ranging from helium ( $Z=2$ ) to thorium ( $Z=90$ ) have been determined from the steady-state solutions of a set of related rate equations that include the effects of electron collisional ionization and excitation, dielectronic and radiative recombination,  $\Delta n=0$  and  $\Delta n \neq 0$  line transitions, and bremsstrahlung. The results for helium ( $Z=2$ ) through molybdenum ( $Z=42$ ) compare very well with other calculations. However, for  $T_e \leq 10\text{keV}$ , the radiation for high  $Z$  elements such as tungsten is significantly higher than previous estimates, by a factor of 10 - 60. The loss rate for high- $Z$  elements indicates that impurity radiation could be responsible for much of the energy loss in tokamaks such as PLT and ORMAK, which have tungsten limiters. Recent experiments have confirmed that tungsten radiation is responsible for 40 - 80% of the energy losses in PLT and 20 - 40% of the energy losses in ORMAK.

Using these loss rates, we have calculated the maximum permitted impurity concentration in a driven tokamak reactor as a function of  $Q$ , the ratio of D-T fusion power to injected beam power, neglecting conduction and convection losses. For  $T_e = 7\text{keV}$ , a  $Q=1$  device could permit a relative tungsten concentration ( $n_W/n_H$ ), of up to  $7 \times 10^{-3}$  whereas an ignition device would require a relative tungsten concentration less than  $2.3 \times 10^{-5}$ . For  $Q \geq 3$ , the critical impurity concentration for different impurity species varies roughly as  $Z^{-2.2}$  to  $Z^{-2.5}$ .

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<sup>\*</sup>Work jointly supported by U. S. ERDA Contracts E(11-1)-3073 and W-7405-ENG-48.

Electron Heat Transport in a Tokamak  
With Destroyed Magnetic Surfaces\*

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Electron heat transport in a sheared magnetic field with destroyed magnetic surfaces has been investigated. It has been shown that a resonant magnetic perturbation  $\delta\vec{B}(\vec{r})$  which destroys surfaces can be divided into an adiabatic part,  $\delta B_a(\vec{r})$ , and a stochastic part,  $\delta\vec{B}_{st}(\vec{r})$ . It is only  $\delta\vec{B}_{st}$  which is responsible for enhanced electron heat transport. In the collisional case, perpendicular electron heat flux  $q_{\perp}$  is given by  $q_{\perp} = \frac{1}{\pi} \frac{1}{R} \kappa_{\parallel} D_{st}(r)$ . Here  $\kappa_{\parallel}$  is the parallel electron heat conductivity,  $D_{st} = \pi R \langle (\delta\vec{B}_{st})^2 \rangle / B_t^2$  is the stochastic diffusion coefficient of field lines,  $B_t$ -toroidal magnetic field and  $R$  is the major radius. The collisionless case as well as time dependent magnetic perturbations have been considered also. Scaling for electron energy confinement time  $\tau_e$  due to this mechanism of heat transport has been proposed.

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\*Work supported by U.S. ERDA Contract No. E(11-1)-3073.

DETERMINATION OF ELECTRON HEAT DIFFUSIVITY FROM THE  
PROPAGATION OF SAWTOOTH HEAT PULSES IN TOKAMAKS<sup>\*</sup>

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A new model is presented in order to calculate the diffusion of sawtooth temperature perturbations observed in ORMAK.<sup>1</sup> The peculiar dipole type perturbations produced by disruptions is shown to give rise to a diffusion coefficient three times smaller than previously estimated.<sup>1</sup> Furthermore, inclusion of geometrical effects results in a substantial improvement in the accuracy with which the electron heat conduction coefficient can be determined from experimental data. The new value obtained from this model is 1.7 times the value obtained from the measured gross energy-containment time.

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<sup>\*</sup>Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation

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<sup>1</sup>J. D. Callen and G. L. Jahns, Phys. Rev. Lett. 38, 491 (1977).

Quasilinear Theory and Enhanced Transport by Lower-Hybrid Waves.\*

J. L. Sperling, General Atomic Company--A general dispersion relation and quasilinear diffusion operator for electrostatic turbulence in magnetized, low-beta, and inhomogeneous (slab model) plasmas is derived. Density and temperature gradients as well as gradients in the streaming velocity parallel to the magnetic field are included in the formalism. Application of the theory to lower-hybrid heating indicates that if there is an RF traveling wave in the poloidal direction at the position of strongest absorption, enhanced radial transport can occur. It appears likely that convective cells can develop and provide a mechanism for converting RF energy into ordered, rather than random, motion. Since this could be detrimental to lower-hybrid heating of magnetically confined plasmas, the conditions for the onset of this process are analyzed in detail. In brief, if the RF traveling wave in the poloidal direction is sufficiently large (e.g.,  $E_p \approx 10^2$  V/cm), the effective heating of tokamak plasmas by lower-hybrid waves may be limited by the anomalous transport processes. Factors which could mitigate the impact of anomalous transport by lower-hybrid waves are absorption in a localized region of space or standing wave patterns in the poloidal direction. For example, if the field strength of the traveling wave component in the poloidal direction is reduced by an order of magnitude, transport is reduced by two orders of magnitude.

\*Work supported by the U. S. Energy Research and Development Administration, Contract EY-76-C-03-0167, Project Agreement No. 38.

CLASSICAL AND ANOMALOUS TRANSPORT IN PLASMA COLUMNS<sup>\*</sup>

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## ABSTRACT

The radial confinement of a plasma column surrounded by a vacuum boundary, axial magnetic field is examined. It has been common to suppose that the plasma diffuses rather slowly across the field lines by means of collisions, and that the rapid current sheath broadening, which occurs during theta-pinch implosions, ceases when the sheath is a couple ion larmor radii in thickness. However, recent studies of the lower hybrid drift instability suggest that anomalously rapid transport can persist even for much broader sheaths,<sup>1</sup> and therefore the effectiveness of radial magnetic confinement for fusion-grade plasmas is called into question.

A solution for the time and space-dependent plasma profile  $\beta(r, t)$  is presented in which regions of classical diffusion (where LHD waves are stabilized by finite  $\beta$  effects<sup>2</sup>) and anomalous diffusion occur simultaneously in different parts of the plasma. The solution is based on a similarity model incorporating regions of classical and anomalous diffusion, in a way that is consistent with the self-similarity.

Several important results follow. The classical region is in the center of the plasma and is separated from the anomalous region (a lower density tail) by a transition region in which the LHD instability is partially stabilized. For plasmas where the column radius-to-ion larmor radius ratio is small, i.e.,  $a/\rho_i \lesssim 3$ , the classical region is small and the radial confinement time can be approximated by a scaling law based on the anomalous diffusivity. For broad plasmas,  $a/\rho_i \gtrsim 10$ , most of the plasma is in the classical region, and the confinement time is nearly classical. For  $3 \lesssim a/\rho_i \lesssim 10$ , the confinement time experiences a rapid transition of two or three orders of magnitude from anomalous to classical.

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\* Supported by Energy Research and Development Administration E(45-1)-2319.

# Theory of Strongly Turbulent Convection of Two-Dimensional Low-Pressure Plasma

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We consider the development of strong turbulence in a low  $\beta$  weakly ionized plasma confined in a strong magnetic field  $B_0 \hat{x}$  subjected to both a gradient in density  $(\partial n / \partial z) \hat{z}$  and an electric field  $\underline{E} = -(\partial \phi / \partial z) \hat{z}$ . The electrons and ions suffer collisions predominantly with the background neutrals such that  $\Omega_e \gg \nu_e$  and  $\Omega_i \lesssim \nu_i$  where  $\Omega_{e,i}$  and  $\nu_{e,i}$  are the respective cyclotron and collisional frequencies. The difference in  $\Omega/\nu$  between electrons and ions gives rise to a mean current density  $J \hat{y}$  resulting from the  $\underline{v}_0 = \underline{E} \times \underline{B} / B^2$  drift of the electrons. This configuration which models the Equatorial Electrojet is unstable if  $\nabla n \cdot (-e \nabla \phi) > 0$ . The unstable fluctuations are nondispersive and therefore weak turbulence theory is inapplicable.

The "direct interaction approximation" of Kraichnan as modified by Kadomtsev is employed to develop a strong turbulence theory which predicts (1) that the power spectrum is isotropic and  $I_k \propto k^{-n}$  (2) that the nonlinear broadening of the frequency spectrum  $\Gamma_k \propto k^2 I_k^{-1/2}$ , (3) that  $n \approx 16/5$  if radar backscatter observations, which indicate that  $I_k \propto v_0^2$ , are taken into account. The mathematical content of our model could represent convective plasma motion in a broad class of large machines for plasma confinement.

Transport Calculations for a Liner Compressed Plasma.\* Ronald E. Waltz, General Atomic Company--Theory and computations for a high- $\beta$  liner plasma are presented. The numerical calculations include cross-field diffusion of magnetic field, heat and impurities; compressional effects; end losses; and radiative losses (including coronal impurity radiation). Both insulating and conducting liner boundary conditions are treated. The formation of a cold dense plasma boundary layer is observed in the former case. A model of impurity boundary layer formation is also given. While impurities are driven out of a straight high- $\beta$  plasma by thermal forces, trace amounts upon injection of the plasma can be troublesome. Solutions to this problem are explored.

\*Work supported by the Electric Power Research Institute (EPRI), Contract RP-921-1.

POSTER SESSION F

# TOPOLOTRON'S UNIQUE, MAXIMAL IMMUNITY TO GLOBAL INTERCHANGE INSTABILITIES

by

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Global interchange instabilities are known to be the "most dangerous" MHD instabilities because, to first order, they cannot be identified by any sensor system and so [Ribe and Rosenbluth] cannot even in principle be feedback stabilized. It is proved rigorously that the TOPOLOTRON magneto-fluid dynamic equilibrium [which, besides having two-dimensional topological stability of its surface magnetic field and three-dimensional topological quasi-stability of its vacuum magnetic field, is, in the sense of topological dynamics the closest possible toroidal analog of an infinite straight theta-tron configuration] is the only possible smooth-surfaced, toroidal, axially-symmetric unity beta magneto-fluid dynamic equilibrium which is [literally] infinitely impervious to the occurrence of global interchange instabilities and which maintains this immunity despite arbitrary small changes in the physical parameters defining the configuration. This result is proved by showing that a TOPOLOTRON is an optimal toroidal average magnetic well in the sense of averaging [Pfirsch and Wobig] appropriate to the application of the BFKK Energy Principle [in Lust and Martensen's equivalent reformulation] to interchange modes. The preceding result is valid for all TOPOLOTRON configurations. For particular configurations, additional stability results have been obtained by application of Martensen's numerical implementation of the Energy Principle, with computer help from INEL (Dr. R. Young) and LLRL. Certain TOPOLOTRON cross-sections [which are non-circular, "kidney-bean"-shaped regions, with the "bumps" nearer the axis of symmetry; cf. Physics of Fluids, vol. 19 (1976), pp. 1810-1819 and IEEE Transactions on Plasma Science, vol. PS-4 (1976), pp. 162-165] have MHD stability for all toroidal modes  $(m,n)$ ,  $m = 1, 2, 3, \dots, N$  and  $n = 1, 2, 3, \dots, N$  where at least  $N = 19$ , for values of beta up to and including unity, and where if FLR effects are included by modification of the Energy Principle according to W. Newcomb's theory of Gyroviscous Fluids then, for sufficiently large aspect ratios, there is complete MHD-FLR stability, i.e.  $N$  is infinity.

UNSTABLE MODES OF A FULLY PINCHED  
RELATIVISTIC ELECTRON BEAM IN A PLASMA\*

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ABSTRACT

We derive and analyze a dispersion relation for the azimuthally symmetric mode of a fully pinched relativistic beam propagating in a uniform plasma. The beam is of low current ( $I_{\text{BEAM}} \ll I_{\text{ALFVÉN}}$ ) and may be compressed in the radial and longitudinal directions. Radial variations of the beam are assumed to be self-similar with mean radius described by an envelope equation.<sup>1</sup> The plasma is considered to be cold and uniform with constant collision frequency. The complete Maxwell's equations are solved including electromagnetic effects. The beam equilibrium profile is taken to be Gaussian in form and the beam is assumed to be charge neutral with no return current. A dispersion relation is derived for perturbations from the equilibrium in the linear regime.

This dispersion relation is studied numerically and analytically. The usual longitudinal beam plasma mode is observed; a small modification of the growth rate due to the finite, pinched character of the beam is calculated. The resistive sausage instability<sup>2</sup> involving transverse oscillations of the beam mean radius is recovered. A new instability, termed the resonant sausage mode, is observed. This mode occurs at  $(\omega, k)$  where both the natural beam sausage oscillations and the plasma oscillations are in resonance; its description involves coupled transverse and longitudinal motions.

Simple expressions for the growth rates  $\omega_i(k_r)$  for these instabilities are found. For a highly relativistic beam the resonant sausage instability has the largest growth rate.

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\* Work performed jointly under the auspices of the U. S. Energy Research and Development Administration, under contract No. W-7405-Eng-48 and the Department of the Navy under Contract number NAONR 11-76.

<sup>1</sup>E. P. Lee and R. K. Cooper, Particle Accelerators, Vol. 7, No. 2 pp. 83-95 (1976).

<sup>2</sup>E. P. Lee "Hose Theory" LLL report UCID 16268 (May 1973).

Compact Experiments for  $\alpha$ -particle Heating\*

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A line of compact experiments (Ignitors), whose main purpose is to study the heating properties and the confinement of  $\alpha$ -particles produced in relatively dense ( $n^0 \geq 10^{15} \text{ cm}^{-3}$ ) D-T plasmas, is proposed. The main goals to be achieved are, in successive stages, the ideal ignition temperature where  $\alpha$ -particle heating balances the power loss by bremsstrahlung, the real ignition conditions, and the power producing phase. For the range of devices studied, the primary heating system is Ohmic and the relevant design parameters are derived from requiring a plasma current  $\geq 3$  MA while the poloidal magnetic can reach values as high as 30 kG and the peak current density 5 kA/cm<sup>2</sup>. Values of the relevant safety factors that are consistent with present day experiments and theoretical expectations on the stability of internal resistive modes in the regimes of interest have been adopted. The first two phases can be attained with  $\beta_p < 1$  by a proper programming in time of the density, current and position of the plasma column. The last phase, where  $\beta_p > 1$ , is attained by magnetic flux conservation and  $\alpha$ -particle heating at ignition so as to preserve the favorable macroscopic stability properties that are established when  $\beta_p < 1$ .

In order to attain the mentioned design parameters, low-aspect ratio and high magnetic field configurations are considered and a variety of compatible auxiliary heating systems has been analyzed. Results for the numerical simulation of the relevant heating cycles, obtained by a numerical transport code that reproduces the observed density and temperature profiles of existing experiments, are presented.

\*Work supported in part by E.R.D.A. and in part privately.

## INTERNAL RING END-STOPPER FOR AN OPEN ENDED DEVICE\*

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A set of ring conductors placed inside a solenoid is proposed as an efficient end-stopper. The rings can stabilize strong end-mirrors by creating a minimum-B situation and via non-adiabatic effects, they tend to trap the end-loss plasma. At the end of a theta pinch they also prevent bad curvature in the central hot plasma region.

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\*Work performed under the auspices of U.S.ERDA.

HEATING AND BOOTSTRAP CURRENT GENERATION IN THE THOR HIGH BETA TOKAMAK EXPERIMENT<sup>†</sup>

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Fast rising magnetic fields have successfully been used to rapidly ( $\sim \mu\text{sec}$ ) heat theta pinch plasmas to thermonuclear conditions. In a new experiment, called Thor, we are investigating the use of these techniques to both heat and produce a high beta tokamak configuration.

The heating technique is a combination of compressional heating and heating due to microinstabilities associated with the diamagnetic current, with some experiments more in one regime than the other. The University of Maryland straight fast theta pinch (FTP)<sup>1</sup> by virtue of its low density has operated mainly in the instability dominated regime. Theoretical modeling<sup>2</sup> of the implosion phase has demonstrated the importance of different instability modes due to the large change in parameters. Typically, in FTP,  $T_i > T_e$  which suggests the importance of modified two stream and lower hybrid drift instabilities. Experimental results suggest the electrons remain cool due to energy transport along the field lines and out of the hot region. For toroidal applications this loss is not expected and consequently the heating may be different. The first phase of the Thor experiment is investigating this problem. In this experiment a low field, high  $q$  ( $\sim 10$ ) initial plasma is excited by a rapidly rising toroidal field (6 kG, 1  $\mu\text{sec}$ ). We have observed<sup>3</sup> rapid ( $\sim 1 \mu\text{sec}$ ) heating of both ions and electrons to keV energies in plasmas with densities  $\sim 1 \times 10^{14} \text{ cm}^{-3}$ , giving  $T_e \gg T_i$ . The hot plasma is highly diamagnetic ( $\beta \approx 1$ ). In addition the toroidal current is increased by a large amount depending on the initial conditions (e.g., initial toroidal current) suggesting some form of bootstrap current effect.

<sup>†</sup>Research supported by Energy Research and Development Administration.

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## The Stability of the Tormac Equilibrium\*

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A unique concept in the design of Tormac is the rotation of the plasma, which is to prevent toroidal drifts from carrying all particles out of the hot region.<sup>1</sup> This work considers the stability of a Tormac-like equilibrium. The plasma is confined in a cylinder with an arbitrary cross section. The magnetic field is in the axial ( $z$ ) direction and rotation takes place in the ( $x, y$ ) plane. A similar system was studied by G. Spies<sup>2</sup> but with a circular cross section. The presence of the flow introduces analytical difficulties as the system is not self-adjoint, and features common to static equilibria have to be investigated anew. The added complexity requires the use of new techniques.

The continuous spectrum is found to be real and to consist of the familiar Alfvén and cusp parts. The singularity of the corresponding "eigenfunction" however is not logarithmic. The question of existence of accumulating unstable eigenmodes is investigated. These are found to exist in the incompressible limit, provided all plasma particles rotate with the same period. The modes are line preserving ( $k = 0$ ), and asymptotically lie on a circle, as in [2]. A necessary condition for stability to stabilize these modes is obtained.

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\* Work support by U.S. ERDA Contract No. EY-76-C-02-3077.

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2. A. Kadish and G.O. Spies, Pulsed High Beta Plasmas, Pergamon Press, 1976. pp. 141-145.

## NEGATIVE ION YIELDS FROM HYDROGEN ATOM BOMBARDMENT OF CESIUM-COATED SURFACES\*

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## ABSTRACT

In an earlier paper<sup>1</sup> we had developed a model for the generation of negative ions by energetic atoms incident upon alkali-coated surfaces. In this paper we report the fractional yield of negative ions per incident atom for hydrogen atom energies in the range one to one hundred electron-volts. The transition probability for electron loss from the negative ion into the metal substrate is calculated using the method of complex eigenvalues. The interaction potential includes a term arising from the image forces and a term due to the electric dipole layer of the alkali coating. The problem is reduced to a one dimensional equation by introducing parabolic coordinates which permits a satisfactory separation of variables. The real and imaginary part of the energy eigenvalue are selected to give a negative ion eigenfunction in the neighborhood of the atom and purely outgoing wave in the asymptotic region interior to the substrate. The total electron loss probability is then an integral over the imaginary part, the integration extending from near the metal outward to infinity.

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\*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

<sup>1</sup>J. R. Hiskes, A. Karo and M. Gardner, JAP 47, 3888 (1976).

## INITIAL STUDIES OF Z-PINCH PLASMA UNDER LINER IMPLOSION CONDITIONS

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## ABSTRACT

We are performing two-dimensional numerical calculations of the dynamic, nonadiabatic behavior of a z-pinch plasma under liner implosion conditions. We use the computer code ANIMAL--A New Implicit Magnetohydrodynamic Algorithm. [1] Only the plasma dynamics are modeled by the code. The cylindrical liner is represented by a rigid, moving wall.

Calculations completed to date use nonequilibrium initial conditions in which the plasma has a uniform density of  $10^{18}/\text{cm}^3$  and uniform temperature of 600 eV. The initial azimuthal magnetic field is uniform 100 kg except for radii less than 3 cm where the field increases linearly with radius. Our calculations show that a 15 cm diameter, 10 cm long plasma imploded at a velocity of 1 cm/microsecond attains an average temperature of 8.1 keV at a liner radius of 0.5 cm; the peak plasma temperature is 19.6 keV. Approximately 59% of the work done by the liner appears as an increase in thermal energy of the plasma. The dominant loss mechanism is thermal conduction to the liner and to the ends of the system. The losses are enhanced by the wall-induced instability first reported by Lindemuth, et al. [2]

We report on the effect of increasing the diameter and length of the system. Comparisons of the two-dimensional, finite length calculations are made with infinite cylinder one-dimensional calculations and "infinite" cylinder two-dimensional calculations which exhibit the wall-induced instability. Typical calculations are displayed in a computer-generated movie.

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## MATERIAL END-PLUGGING OF LINEAR THETA PINCHES\*

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The interaction of a theta pinch plasma with material end-plugs has been investigated numerically. The effects of hydrodynamics, electron and ion thermal conduction, and electron-ion energy exchange are included. Calculations of open-ended theta pinches with and without mirror fields have also been made and compare well with experimental data. It is shown that material end-plugs can give a very appreciable improvement in energy confinement for the short systems considered. Plasma recompression driven by the material ablated from the plug is significant and could provide a useful plasma diagnostic. Radiation processes are also considered.

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\*Work performed under the auspices of U.S.ERDA.

Application of Powerful Millimeter Wave Sources  
to the Tokamak Program

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This paper<sup>1</sup> is to acquaint the tokamak community with recent work at the Naval Research Laboratory and in the Soviet Union on the development of a new class of microwave tubes based on cyclotron masers. One type of tube, based on thermionic cathode technology, is capable of generating tens of kilowatts at wavelengths above one millimeter with high ( $\sim 50\%$ ) efficiency in either a pulsed or CW mode. Another type, based on relativistic electron beam technology, is capable of much higher power (several megawatts), but low ( $\sim 1\%$ ) efficiency, and it delivers its energy in 50 nanosecond pulses. Applications to be discussed are electron-cyclotron heating, local electron cyclotron heating, pre-ionization, test wave launching, ion temperature measurement, and effective trapped-electron collision frequency measurement.

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<sup>1</sup>Development of High Power Millimeter Wave Cyclotron Masers at NRL and Its Relevance to CTR, Naval Research Laboratory Memorandum Report 3493, 1977, W. Manheimer and V. Granatstein.

Supported by ERDA Contract E(49-20)-1006.

CONFINEMENT OF NON-NEUTRAL PLASMA IN A TORUS WITH APPLICATIONS TO HIGH INTENSITY PARTICLE ACCELERATORS,\* A. Mondelli and N. Rostoker, Maxwell Laboratories, Inc., and University of California, Irvine.

Electrons with their large  $e/m$  may be confined in a torus with a toroidal magnetic field. The self-electric field of the electrons provides a rotational transform. For energetic ions with small  $Ze/M$ , the magnetic field has little effect. However, the electric field within the toroidal electron cloud provides strong confinement/focusing for ions as long as the number of ions is much less than the number of electrons. Toroidal experiments at Maxwell Laboratories have shown that almost all of the confined electrons are in trapped orbits and do not go around the torus.<sup>1</sup> The electron magnetic moment  $\mu_e > \mu_i/Z$  where  $\mu_i$  is the ion magnetic moment and  $Z$  is the charge. Thus a toroidal electric field of suitable magnitude can be applied so that it accelerates the ions around the torus, but does not accelerate most of the electrons. Based on these physical principles, very compact toroidal accelerators can be designed. We describe accelerators to produce several megajoules/pulse of 100 GeV heavy ions or 2.5 GeV protons. These accelerators involve a major radius of 3 - 5 meters and a minor radius of a few centimeters. Analysis and control of magnetron, diocotron, ion resonance, loss cone and two-stream instabilities will be described.

\* Supported by Defense Nuclear Agency and National Science Foundation.

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Evolution of a z-pinch as revealed by a 3-d, e-m particle code.\*

D. NIELSEN, J. GREEN and O. BUNEMAN, Stanford University--A z-pinch has been simulated in a (relativistic) 3-d, e-m particle code. A steady E-field was applied along a plasma column with diffuse boundaries held by a moderate parallel guiding B-field. After an initial uneventful acceleration phase the column suddenly develops a helical large-scale, fluid-like  $m=1$  mode when the magnetic self-fields become comparable to the applied magnetic field. Eventually while the electrons become more and more relativistic, the bulk of the plasma contracts into a helix. A movie of this sequence will be shown and theoretical interpretations of each phase will be attempted.

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# A Diffuse Profile Topolotron Plasma Configuration

by

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A diffuse profile, axially symmetric solution to the Grad - Shrafranov (GS) equation

$$\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = - \frac{4\pi r^2}{r_0^2} \frac{\partial p}{\partial \psi} - \frac{1}{2} \frac{\partial I^2}{\partial \psi}$$

is obtained in principle for a topologically stable topolotron configuration. The method of solution consists of choosing coordinates such that the plasma surface corresponds to a level surface of the coordinate system. The (GS) partial differential equation then reduces to a system of forced ordinary differential equations. Finally, a set of truncated solutions to this system is forced to satisfy a particular choice of topolotron boundary conditions by a least-squares minimization technique. A graph of the plasma cross section in terms of constant  $\psi$  contours will be displayed. These curves exhibit the characteristic kidney bean shape of the topolotron equilibrium and consist of two distinct nested sets with a separatrix between.

# ELECTRON ENERGY AND SPACE CHARGE CALCULATIONS IN REFLEX DIODES\*

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## ABSTRACT

Previously reported Monte Carlo code calculations<sup>1</sup> of the electron energy distributions and the consequent reflex triode characteristics will be presented for two different anode designs. In addition, a generalized formulation of Poisson's equation will be used to examine the virtual cathode side of a reflex diode. The familiar "resonance" solution for the reflex triode is again found, but with a different physical interpretation. In the former case the current diverges, but in the virtual cathode space the linear dimension diverges as one approaches the "resonance."

\*Work performed under the auspices of the U.S. Energy Research and Development Administration, under contract No. W-7405-Eng-48.

<sup>1</sup>J. W. Shearer, Bull. of APS 21, 1030 (1976).

State of Stellarator Research<sup>\*</sup>

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Stellarators have recently been shown to be extremely effective toroidal magnetic confinement devices, with high energy confinement times flat density and temperature profiles and  $q$ 's that can be of the order of  $1/2$ . In addition, currentless operation is effective and permits the establishment of the confining fields before plasma is introduced. No disruptive instabilities have ever been reported by present day ohmically heated stellarators when the helical rotational transform is large enough to outweigh the current rotational transform. 23 Stellarators are currently in operation world-wide.

Stellarators, and their topologically similar relatives, torsatrons, are toroidal devices in which the rotational transform is produced by means of currents flowing in conductors external to the plasma. Ohmic heating is not necessary for equilibrium. The non-axisymmetries in such devices result in additional classes of trapped particles with a theoretical prediction of diffusion in the trapped-particle regime greater than for an axisymmetric device, although such enhanced diffusion has not been seen experimentally.

Recent fully non-axisymmetric calculations of the MHD equilibrium of both circular and non-circular cross section stellarators show that betas of a few percent can exist.

Energy containment time of stellarators compare favorably with those of equivalent sized tokamaks. If necessary, stellarators, and more importantly torsatrons, can be operated with built-in poloidal divertors. The latter device requires no external toroidal field coils and half the number of helical conductors of a conventional stellarator. Figure-8 stellarators require no helical conductors. All stellarator type devices can, in principle, be operated in steady state. Both neutral injection and RF heating can be used in currentless stellarators, the latter being very effective, yielding parameters equivalent to those produced by Ohmic heating.

<sup>\*</sup> Work supported by the National Science Foundation under Grant ENG 75-11168

High-Frequency Stability of a Field-Reversed  
Ion Ring in a Background Plasma\*

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The high frequency ( $\omega \gg \Omega_i$ ) flute-like electrostatic normal modes are found for a long thin field-reversed ion ring immersed in a much denser, colder, uniform background plasma. Instabilities of the weak beam type are found at frequencies somewhat above the lower hybrid frequency. For a sufficiently high energy, low density beam (not difficult to satisfy with reasonable beam and plasma parameters), most of these modes are stabilized by electron cyclotron damping near the surface of field reversal. A few unstable modes remain, localized near the center of the beam. The growth rate of these modes is lower for higher beam energies, fatter rings, and greater degrees of field-reversal. For reasonable parameters, the maximum growth rate can be as low as a few percent of the ion cyclotron frequency.

\*Work supported by the National Science Foundation Grant Number DE 575-02797 and ERDA Contract Number E(11-1)3170.

## HEAVY ION INDUCED INERTIAL FUSION: THE BEAM PROPAGATION PROBLEM

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We explore the processes occurring when a beam of energetic ions is transmitted through a large vacuum chamber to strike a small pellet. Because of repetition rates, pumping and cooling speeds, etc., the chamber is expected to contain a rather cool diffuse plasma, and the interaction between the beam and this background lead to a number of phenomena: neutralization of charge and current; electron and ion drag; the subsequent reappearance of current, and a host of instabilities, in spite of which the propagation problem does not appear to rule out this approach to fusion.

<sup>\*</sup>Supported by NSF Grant PHY-75-02690.

<sup>\*\*</sup>Partially supported by NSF Grant PHY-75-02690.

OPERATING POINT CONSIDERATIONS FOR THE  
ELMO BUMPY TORUS REACTOR (EBTR)\*

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The EBT confinement concept provides an attractive alternative approach for a fusion power system. The EBT has several advantages which distinguishes an EBT system from the other approaches. These are a potential for high beta -- perhaps as large as 40% as compared with about 10% in tokamaks --, high power density, large aspect ratio, and steady-state (non-pulsed) fields. A uniform and self-consistent treatment of the EBT reactor scaling and design has been studied and reference design has been selected.<sup>1</sup> Power production, technology, and engineering considerations were used to establish the EBTR characteristics and operating points. Refinements in plasma engineering and plasma physics have resulted in a revised system description. The plasma size and parameters are determined self-consistently from theoretical models under reasonable assumptions which show they are adequate from a containment point of view. The effect of the different scaling assumptions on EBTR performance and operating point have been analyzed. Although in the present EBT device (EBT-I) plasma transport seems to be governed largely by neoclassical processes, hypothesized anomalous transport losses are included into the plasma simulation model in addition to the neoclassical and/or classical transport in order to test the sensitivity of EBTR parameters. The selected scaling parameters are shown to be flexible enough to permit any physically acceptable departures from neoclassical and/or classical confinement. For example, for the particular case studied, it is shown that the reference EBT reactor can accommodate any anomalous losses fifteen to twenty times greater than the neoclassical and/or classical losses at the operating point (steady-state). Results will be discussed.

\*Research sponsored by the Energy and Development Administration under contract with the Union Carbide Corporation.

<sup>1</sup>D. G. McAlees, N. A. Uckan, et. al., ORNL/TM-5665, Oak Ridge (November, 1976).  
N. A. Uckan, et. al. "The ELMO Bumpy Torus Reactor (EBTR) Reference Design";  
and "Plasma Engineering Aspects of the ELMO Bumpy Torus Reactor (EBTR)"  
(submitted to Nuclear Fusion).

## Orbit Calculation for Field Reversal Mirror\*

by

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With the recent achievement of high beta in 2X-II<sup>(1)</sup>, the concept of a field reversed mirror (FRM) created by internal plasma currents has gained in interest. In contrast to the Astron concept, the FRM utilizes lower energy neutral beam injection, thereby allowing a smaller device<sup>(2)(3)</sup>. In the present work we study particle orbits for the FRM based on an analogy with the spherical vortex configuration originally developed by M. J. Hill<sup>(4)</sup>. With this method analytic solutions are obtained. For motion in the midplane, the orbits can be written in terms of 1st and 3rd kind elliptical integrals. Requirements for absolute confinement are obtained, and four distinct classes of orbits are identified that are bound by limits on the canonical angular momentum  $P_\theta$ . The improvement in confinement over a simple mirror as well as natural divertor properties of the FRM are shown by these results. Implications of the orbit classification to diffusion flow are under study. It is thought that an "effective" diffusion coefficient can be derived using these results, and this can in turn be used in reactor design studies.

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\*Work supported by ERDA.

- (1) G. Logan, Lawrence Livermore Laboratory private communication, February 1977.
- (2) W. C. Condit, et al., "Design Calculations for a Field-Reversed Mirror Reactor," 2nd ANS Topical Meeting on Fusion Technology, Richland, WA (1976).
- (3) G. H. Miley and D. Driemeyer, "Approach to Small-Size Field-Reversed Mirror Reactors," Bull. Am. Phys. Soc., Ser. II, 21, 1162 (1976).
- (4) M. J. Hill, phil. Trans. A., C/XXXV (1894).

## ALFVEN-ION CYCLOTRON INSTABILITY IN THE CENTRAL CELL OF TMX\*

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## ABSTRACT

The central cell of TMX<sup>1</sup> may require hot ion injection.<sup>2</sup> Hot ions are confined magnetically, not electrostatically,<sup>3</sup> and so manifest a certain irreducible (though slight) anisotropy. This slight anisotropy leads to AIC instability.<sup>4</sup> We show that at such low anisotropy the instability is not absolute but convective, and satisfies the Rosenbluth criterion<sup>5</sup> that fewer than seven amplitude exponentiations take place in the length of the cell. The ion velocity distribution is modelled by superposing a hot Maxwellian with a loss-cone on a cool Maxwellian which is isotropic.

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\*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

<sup>1</sup>TMX MAJOR PROJECT PROPOSAL, F. H. Coensgen et.al, LLL-Prop-148.

<sup>2</sup>Ibid, p. 14.

<sup>3</sup>Ibid, p. 3.

<sup>4</sup>J. G. Cordey and R. J. Hastie, Phys. Fluids 15, 2291 (1972), and references therein; also L. D. Pearlstein, D. C. Watson, L.L. Lodestro, in Proc. Ann. Meeting on Theor. Aspects of Controlled Thermonuclear Fusion, Madison, 1976.

<sup>5</sup>M. N. Rosenbluth and R. F. Post, Phys. Fluids 8, 567 (1965).

POSTER SESSION G

PARTICLE ORBITS IN SHEARED MAGNETIC FIELDS  
AND DRIFT WAVE INSTABILITIES.\*

G1

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Particle orbits in sheared magnetic fields are studied, analytically using operator techniques<sup>1,2</sup> and also through analytical or numerical integration of the equations of motion. The time evolution, given by

$$\underline{r}(\tau) = e^{\tau L} \underline{r} e^{-\tau L}$$

is explicitly evaluated for simple field configurations, and tackled non-perturbatively by using operator algebra and Zassenhaus expansions. We compare the orbits for the commonly used field models  $\underline{B} = B_0(0, -\sin\theta(x), \cos\theta(x))$  and  $\underline{B} = B_0(0, -X/L_S, 1)$ , and also consider variable  $B_0$  and variable shear parameter  $S = d\theta/dx$ . Our objective is to understand the effects of detailed orbits on the stability criteria of various drift modes.<sup>3</sup> Earlier treatments in the literature have essentially used rather simplified orbits, which in some aspects differ substantially from the exact orbits. While the small  $\rho k_\perp$  domain is not much affected, modes with wavelengths comparable to or smaller than the Larmor radius are significantly modified by these orbital effects.

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\* Work supported by U. S. Energy Research and Development Administration.

- <sup>1</sup> P. Bakshi and W. Bellew, "Shear Stabilization of Drift Waves: A New Approach", AFCRL-TR-74-0619 (1974)
- <sup>2</sup> P. Bakshi, "A New Formalism for Microinstabilities in the Tormac Sheath Regime", COO-2714-2 (1976).
- <sup>3</sup> W. Bellew and P. Bakshi, Bull. Amer. Phys. Soc. 21, 1122, (1976)

The Dissipative Trapped-Electron Mode  
in Vertically Elongated Tokamaks

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The role of the dissipative trapped-electron mode in a vertically elongated tokamak is studied. We consider in particular the following two aspects--the effect of the absence of poloidal uniformity on the shear-induced damping rate and the effect of slowed-down or reversed electron toroidal drift on the growth rate. The poloidal mode structure is calculated for a vertically elongated ellipse assuming no shear. The full dispersion relation is the determinant of an infinite matrix, each element of which is an infinite sum consisting of plasma dispersion functions which result from the ion orbits. The dispersion relation is solved numerically by truncating the determinant at the point of convergence (typically  $7 \times 7$ ). It is shown that vertical elongation can significantly reduce the maximum growth rate of the trapped-electron mode.

The problem of shear induced damping neglecting trapped electrons is governed by a two-dimensional partial differential equation which is solved analytically under the assumption that mode rational surfaces are decoupled. It is shown that the shear-induced damping rate is insensitive to the amount of vertical elongation.

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# AN IMPROVED ESTIMATE OF THE TRAPPED ION MODE ENERGY LOSS FROM TOKAMAK REACTORS\*

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It is usually thought that theoretical estimates of the energy loss due to trapped ion mode (TIM) turbulence provide the most stringent conditions for the attainment of ignition in tokamak reactors. In estimating the  $n\tau$  of a tokamak reactor subject to trapped ion modes, it has been customary<sup>1</sup> to replace the variables in the transport coefficient  $D_{TIM}$  by average values and to obtain the energy containment time from  $\tau = a^2/4D_{TIM}$ .

In this paper we will show that the simple estimate of  $n\tau$  described above is inappropriate for nonlinear diffusion processes such as trapped ion turbulence. A correct estimate is obtained by solving self-consistently the particle and energy transport equations including particle and energy sources. An approximate analytic solution to the problem is presented which yields a simple formula that can be used in zero-dimensional reactor design calculations. It is found that the parameter dependence of  $n\tau$  is preserved but its magnitude can exceed the conventional estimate by one to three orders of magnitude depending on the method of fueling used. The physical reason is that  $D_{TIM} \propto r^{5/2} T^{7/2} (dn/dr)^2$ . Edge fueling flattens the density profile<sup>2</sup> and the self-consistent  $T$  is such that this product never approaches the value obtained by replacing the variables by their average values.

As a result of these calculations, we conclude that transport due to trapped ion mode turbulence will allow ignition of one meter radius plasmas at moderate magnetic fields; in fact, trapped ion mode turbulence can prove beneficial in providing a convenient method for controlling plasma reactivity.

\*Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>1</sup>H. P. Furth, Nucl. Fusion 15, 487 (1975).

<sup>2</sup>J. R. McNally, Jr., J. F. Clarke, R. H. Fowler, T. Hiraoka, R. D. Sharp, D. Steiner, IAEA-CN-33/G 1-3.

## 2D NONLINEAR SATURATION OF THE TRAPPED-ION MODE BY MODE COUPLING\*

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We present a study of the nonlinear saturation by mode coupling of the dissipative trapped-ion mode in which both radial and poloidal variations are considered. The saturation mechanism consists of the nonlinear coupling via  $\underline{E} \times \underline{B}$  convection of energy from linearly unstable modes to stable modes. Stabilization is provided by Landau damping from trapped and circulating ions at short poloidal wavelengths, by ion collisions at long poloidal wavelengths, and by finite ion banana-width effects at short radial wavelengths. Beginning with the Kadomtsev-Pogutse fluid equations, we extend a nonlinear one-dimensional partial differential equation for the electrostatic potential to two-dimensions and to third order in amplitude. Included systematically are kinetic effects in linear approximation, e.g. Landau damping and ion banana-width excursions.

Steadily propagating nonlinear wave forms with only a poloidal variation are found to be stable to perturbations with poloidal variation,<sup>1</sup> but are unstable to perturbations of short radial wavelength. This motivates a fully two-dimensional analysis which includes nonlocal effects due to magnetic shear. We have analytically obtained simple time-dependent and steady-state solutions describing the linear and nonlinear stages of the instability. For systems which are far from linear marginal stability, but have a sufficiently narrow spectrum of unstable modes, saturated states can be established by three-wave interactions. This can potentially result in significantly reduced saturated amplitudes and associated particle transport as compared with the predictions of one-dimensional theory<sup>1</sup> and the Kadomtsev-Pogutse transport estimate. Very close to linear marginal stability, the resonant four-wave interaction of a single unstable mode with itself can account for saturation. Stable nonlinear equilibria are found with amplitudes scaling as the square root of the linear growth rate and particle transport scaling directly as the growth rate.

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\*This work was supported by ERDA Contract Nos. E(11-1)-3073 and W-7405-Eng-48.

<sup>1</sup>B. Cohen, J. Krommes, W. Tang, and M. Rosenbluth, Nucl. Fusion 16, 971 (1976).  
[We request the poster format, B. Cohen — (415) 447-1100 Ext. 4735.]

# A Derivation of the Dissipative Trapped Electron Instability\*

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A spectral representation of the drift kinetic equation in toroidal geometry yields the trapped electron response to a perturbation potential in terms of the equation

$$\frac{\partial}{\partial x} \frac{1}{\bar{\omega}^2 - x^2} \frac{\partial f_{m,n}}{\partial x} + f_{m,n} = G_{m,n},$$

where

$$G_{m,n} = 2^{-1/2} \frac{e}{T_e} f^0 \left( \frac{\bar{\omega} - \bar{\omega}_{*e}}{\bar{\omega}} \right) \frac{\partial}{\partial x} \left( \frac{\varphi_{m+1,n}}{\bar{\omega} - x} - \frac{\varphi_{m-1,n}}{\bar{\omega} + x} \right).$$

Here  $f_{m,n}(x)$  and  $\varphi_{m,n}$  are respectively the perturbed trapped-electron distribution function and the potential,  $m$  and  $n$  are the poloidal and toroidal mode numbers for a mode nearly perpendicular to  $B_m$ ,  $x = 2^{3/4} \epsilon^{1/2} \cos \alpha$ ,  $\epsilon = r/R_0 \ll 1$  is the ratio of minor to major radii,  $\alpha$  is the pitch angle of the velocity relative to  $B_m$ ,  $\bar{\omega} = (\omega + i\nu_{ef})/\omega_b$ ,  $\bar{\omega}_{*e} = (\omega_{*e} + i\nu_{ef})/\omega_b$ ,  $\omega$  is the frequency,  $\nu_{ef}$  is the "effective" collision frequency,  $\omega_b$  is the bounce frequency,  $\omega_{*e}$  is the "electron" drift frequency and  $f^0$  is the Maxwellian distribution.

This equation which also describes electron trapping in a large amplitude Langmuir wave\*\*, may be solved by expansion in parabolic-cylinder functions  $\psi_j(x)$  to give  $f_{m,n} = G_{m,n} - \sum_j \beta_j (d\psi_j/dx)/(\bar{\omega}^2 - 2j - 1)$ , where  $\beta_0 = 0$  and  $\beta_1 = 3.76(e f^0/T_e)(\varphi_{m+1,n} + \varphi_{m-1,n})(\bar{\omega} - \bar{\omega}_{*e})/\bar{\omega}$ . Integration of  $f_{m\pm 1,n}$  yields the trapped electron densities of the satellite modes,  $n_{m\pm 1,n} = -1.4 e^{1/2} (e/T_e)(\varphi_{m+1,n} + \varphi_{m-1,n})\langle(\bar{\omega} - \bar{\omega}_{*e})/\bar{\omega}\rangle$  and the dissipative trapped electron modes appears as a result of the coupling between these satellites by the motion of trapped particles.

\* Work supported by ERDA contract EY-76-S-02.2200.\*000

\*\* L. M. Al'tshul' and V. I. Karpman, Sov. Phys. JETP 22, 361 (1966)

# Effect of Random Potential Fluctuations on Drift Waves<sup>\*</sup>

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We have investigated the effect of low-frequency long-wavelength potential fluctuations (due to conventional convective cells or drift convective cells<sup>1</sup>) on the stability of a drift wave. \* Considering the polarization drift nonlinearity to be the dominant nonlinearity and treating the potential fluctuations due to the drift wave as a perturbation on the background noise, we have derived a linear stochastic differential equation for the coupling between the drift waves and the background turbulence. Using Kellar's method, we introduce a closure by retaining terms up to the second order in the perturbed field. The equation for the average amplitude then gives a modified dispersion relation for the drift waves in the presence of the turbulent background. By choosing a Gaussian spectral distribution for the noise, we derive an analytic expression for the modified growth rate for the drift waves and obtain a reasonable level of turbulence for the stabilization of the drift waves when  $k_{\perp}L > 1$  where  $k_{\perp}$  is the wave vector after drift wave and  $L$  the correlation length of the background turbulence. This seems to be consistent with some recent measurements in a collisional plasma in the FM-1 device.<sup>2</sup>

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<sup>\*</sup>Work supported by U. S. AFOSR Contract No. F44620-75-C-0037.

1. S. M. Mahajan, P. N. Guzdar, P. K. Kaw and C. Oberman, Sherwood Meeting 1977.
2. M. Okabayashi and V. Arunasalam, Princeton Plasma Physics Laboratory PPPL-1310 (1976) unpublished.

# Poloidal Structure of the Trapped-Electron Modes Including Curvature Drifts

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We address the problem of determining the detailed field-aligned poloidal structure of the dissipative trapped-electron modes (DTEM's) including magnetic curvature terms for both electrons and ions. As in a previous study,<sup>1</sup> we determine the amount of ballooning by Fourier analysis. The electron curvature drifts enter in orbit-average form; i.e., independent of the poloidal angle, and involve no additional complication in the structure calculations. In contrast, because of the low thermal ion bounce frequency and finite ion gyroradius effects, the ion curvature drifts considerably complicate the velocity space integrals in obtaining the perturbed ion density response.

We have numerically evaluated the resulting linear dispersion relation using optimistic PLT parameters. Including the ion curvature drifts allows two unstable branches. One branch is decreased in wave frequency. It has a negative ballooning ( $\cos \theta$ ) coefficient and negligible growth rates. The more important branch has an increase in wave frequency and increased ballooning. The growth rates are approximately equal to those found using the electron drifts alone. At low values of  $k_{\theta} \rho_i$ , this mode reduces to a damped ion acoustic mode.

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<sup>1</sup>P. C. Liewer, W. M. Manheimer and W. M. Tang, Phys. Fluids, 19, 276.

## Nonlinear Effects in Drift Waves\*

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## Abstract

The nonlinear drift wave equation formulated from fluid description is studied both analytically and numerically. This fluid description is three dimensional and contains the nonadiabatic, dissipative electron-wave interactions that produce instability in the waves.

In the case when coherent motions are assumed, the nonlinear equations have a solitary wave solution localized in the direction of propagation and nonlocalized in the direction of the gradient. Petviashvili shows and we observe that this mode is unstable to self focusing in the  $x$ -direction. We find that for two dimensional localized solutions there are two classes of the finite amplitude solutions, (1) the solutions studied by Petviashvili and (2) the finite amplitude continuation of the small amplitude oscillatory drift waves. Analytical results including the characteristics of the two solutions, existence of the energy integral and the resulting effective potential will be reported.

When many small amplitude, weakly correlated drift waves are assumed, we make use of the random phase approximation to derive the balance equations for the spectral distribution  $I(k_y)$ . An introduction of the technique to project out the radially localized eigenmode components from the general turbulent field is shown to be necessary to derive the 2D mode coupling equation. Approximate stationary solutions for the spectral distribution of the potential fluctuations are obtained from the resulting balance equations. Results of numerical simulations based on the nonlinear drift wave equation described above will be reported. The equation contains the nonlinear  $E \times B$  and polarization drifts of the plasma.

\*This work supported by U.S. ERDA Contract EY-76-C-05-4478.

# Microscopic Magnetic Islands Due to Alfvén Instability in a Nonuniform Plasma<sup>\*</sup>

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Traditionally, anomalous transport from a magnetically confined plasma is ascribed to the presence of electrostatic drift wave fluctuations, which are driven to large amplitudes by the expansion free energy in the plasma. A mechanism of equally great interest, which seems to have received less attention, is the excitation of low frequency short wavelength, predominantly electromagnetic fluctuations by the same free energy source. These latter fluctuations may produce local "tearing" and break-up of magnetic surfaces and thus essentially permit transport along newly "reconnected" field lines. This process is especially significant for short wavelength fluctuations, since the mode-rational surfaces are densely packed and considerable overlapping "magnetic islands" may readily occur. It is well known that in a finite- $\beta$  plasma, Alfvén waves are coupled to drift waves. In the local analyses, Alfvén waves are known to be unstable when the electron diamagnetic drift frequencies  $\omega_e^*$  are greater than the Alfvén frequencies  $\omega_A$ ; which is satisfied, nonlocally, near the mode rational surfaces. We present, in this work, radial eigenmode analyses of Alfvén waves using a matching procedure and a correct ordering scheme. The results show the Alfvén mode has a growth rate  $O(1/\beta)$  larger than that of the drift wave. Furthermore, there exists a finite tearing component ( $\delta b_x \neq 0$ ) at the mode rational surface; which can lead to formation of microscopic magnetic islands. For  $\beta \sim \delta n/n \sim 10^{-2}$ , the island width is estimated to be of order of ion Larmor radius. Estimates of anomalous transport coefficients are also obtained.

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<sup>\*</sup> Work supported by U. S. ERDA Contract No. E(11-1)-3073.

# A Finite- $\beta$ Simulation Model for Low-Frequency Instabilities\*

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We have developed a  $2\frac{1}{2}$ -D particle simulation code suitable for studying low-frequency instabilities in a finite- $\beta$  plasma where self-consistent electromagnetic fields are important. The code is based on the nonradiative Darwin formulation of Maxwell equations together with the guiding center approximation for the electrons. For a low- $\beta$  plasma, the self-consistent magnetic field along the direction of the applied external field  $B_0$  can be neglected. Furthermore, by assuming  $k_{\parallel} \ll k_{\perp}$ , only the component along the magnetic field of the transverse current is kept in Ampere's law. For the particle-pushing, a predictor-corrector scheme has been used for both the ions and the electrons. The parallel electric field has been calculated by properly taking into account the "shielding" effect from the electron inertia term, similar to that described in Ref. 1.

For a homogeneous plasma, test results have shown excellent agreement with the theoretical predictions in terms of the characteristic frequencies, in particular, the low-frequency shear Alfvén waves and the lower hybrid oscillations. Total energy conservation is also good.

This code is now being used for the study of the finite- $\beta$  effects on the low-frequency microinstabilities with the realistic mass ratio. Coupling of the drift waves with the tearing modes will also be investigated using the present model. Extension to fully three dimensional model is straightforward and is now underway.<sup>2</sup>

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\*Work supported by U. S. ERDA Contract No. E(11-1)-3073.

<sup>1</sup>Busnardo-Neto and et al., J. of Comp. Phys. 23, 300 (1976).

<sup>2</sup>C. Z. Cheng and H. Okuda, Princeton Plasma Physics Laboratory PPPL-1310 (1976).

# Nonlinear Saturation of the Trapped Electron Instability

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We have analyzed the threshold saturation behavior of the trapped electron instability for conditions corresponding to those of the Columbia Univ. linear mirror experiment.<sup>1</sup> The trapped electrons are described by a drift-kinetic equation with a collisional Krook term. The "circulating" electrons obey a Vlasov equation while the ions are treated via a viscous fluid equation. We establish a linear threshold resulting from balance between the destabilizing trapped electrons and the ion viscosity at a particular electron collision frequency  $\nu_0$ . We then use nonlinear analytic methods developed earlier<sup>2</sup> to calculate the saturation amplitude and frequency shift of the threshold eigenmode as a function of  $(\nu - \nu_0)/\nu_0 \equiv \Delta$ . The onset is found to be of the "soft" variety. The predicted amplitude of the fluctuating electric potential  $\phi$  is  $|\Gamma|^2 \approx 0.05\Delta$  where  $\phi$  is in units of  $KT_e/e$ . This is in good agreement with the observation. A net frequency shift,  $\delta\omega/\omega_0 \approx -1.5\Delta$  (from the threshold value  $\omega_0$ ) is also predicted. The mechanism of nonlinear saturation, as determined by finding the dominant term in the saturation condition, appears to be related to flattening of the circulating electron distribution function.

1. S. C. Prager, T. C. Marshall and A. K. Sen, Plasma Phys. 17, 785 (1975).
2. A. Simon and M. N. Rosenbluth, Phys. Fluids 19, 1567 (1976).

Effects of Shear and Density Profile on the Spatial Structure of the  
Trapped Electron Mode \*

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Abstract

It is shown that drift wave eigenmodes in a torus in the trapped electron regime have a global radial structure, as opposed to the shear-localized modes obtained in the slab model. The global structure is due to the strong coupling between the poloidal Fourier components from neighboring rational surfaces. The mode is found to rotate in order to align itself with the magnetic field. In the absence of macroscopic profile effects, e.g., variations of the diamagnetic frequency with radius, an infinite radial extent is implied by the structures obtained when the number of poloidal components  $\left(\varphi_m(r) \exp(im\theta)\right)$  in the calculation is varied. It should be noted that the stabilizing effect of shear remains in the truncated system. Since the mode has a global radial structure, spatial variations in the macroscopic parameters must be considered. In the simplest case, in which the electron and ion temperatures are constant, the spatial variations appear only in the diamagnetic drift frequency. For a density profile which allows a maximum in  $\omega^*$ , the modes become localized and the stabilizing effect of shear reduced. More realistic profiles are discussed qualitatively.

\*This work supported by U.S. ERDA Contract EY-76-C-05-4478.

## A Physical Interpretation of Dissipative Drift Instabilities

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Electron collisions are responsible for many instabilities in an inhomogeneous plasma, e.g., the dissipative trapped electron mode (DTE) and the dissipative trapped ion mode (DTI). These instabilities are of particular interest because they cause the transport of particles and energy. We present a simple physical explanation of these instabilities, based on fundamental principles, which emphasizes the connection between wave growth and particle transport. Conservation of momentum is invoked to obtain the relation

$$\dot{\mathcal{P}}_y = -(eB/c)(\Gamma_e - Z\Gamma_i) \quad (Ze \text{ is the ion charge}) \quad (1)$$

where  $\dot{\mathcal{P}}_y$  is the rate of change of the wave momentum, and  $\Gamma_e$ ,  $\Gamma_i$  are the fluxes of electrons and ions associated with the wave. The wave momentum is related to the wave amplitude,  $\phi$ , by

$$\mathcal{P}_y = k_y (\partial \epsilon / \partial \omega) (k^2 \phi^2 / 8\pi) \quad (2)$$

The particle fluxes are estimated by using a random walk model in which the motion of the particles in the wave field determines the step size, while the collision frequency determines the correlation time. These estimates, together with Eqs.(1) and (2), are used to determine the linear growth rates, for both the DTE and the DTI. The usual linear growth rates are recovered.

This viewpoint illuminates the destabilizing effect of electron collisions, the stabilizing effect of ion collisions, and the relation between the "quasi-linear" particle flux and the linear growth rate. Momentum conservation can also be invoked in the non-linear regime. In particular, we conclude that, if no ion dissipation mechanism is involved, then the electron particle flux must vanish when the wave spectrum reaches a steady state.

Adiabatic and Nonadiabatic Particle Motions,  
Resonance Overlapping and Stochastic Diffusion\*

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We have considered the motion of an electron in the field of two plane waves. Such motion exhibits an interesting variety of adiabatic and nonadiabatic behavior. In particular we have studied such nonadiabatic motions which results in particle diffusion in the velocity space. We have shown that the "resonance overlapping criterion" which is frequently used to describe the transition to diffusion like motion is not always the correct one. A more accurate condition is proposed. The transition between adiabatic and nonadiabatic regimes and velocity space diffusion in the presence of many waves has been also studied numerically and compared with the theoretical prediction. We have shown that the stochastic nature of nonadiabatic motion has nothing to do with the random phase approximation, which is often thought to be a necessary condition for the quasilinear description.

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ON THE CONTRIBUTION OF THE DISSIPATIVE TRAPPED  
ELECTRON INSTABILITY TO THE ENERGY BALANCE IN TFR.

TFR Group\*

ASSOCIATION EURATOM-CEA SUR LA FUSION  
*Département de Physique du Plasma et de la Fusion Contrôlée  
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TFR results are analyzed through the *looking-glass* of the trapped particle instabilities. The measured anomalous electron and ion transport are compared to the predicted values.  $T_e$  profiles with the plasma parameters evolution are compared to the profiles computed on the basis of a marginal stability analysis.

\* Submitted by C. RENAUD

# Integral Formulation for the Two-Dimensional Spatial Structure of Drift and Trapped-Electron Modes\*

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Over the past year, a major effort has been underway to increase the accuracy and generality of calculations of the linear growth rate and two-dimensional spatial structure of trapped-electron and drift modes. Results in the limit  $k_r \rho_i \ll 1$ , where  $k_r$  is the radial wavenumber and  $\rho_i$  is the ion gyro-radius, were reported in Ref. 1, and the generalization for arbitrary  $k_r \rho_i$  was presented briefly in Ref. 2. The method used involves expansion of the perturbed electrostatic potential in complete sets of radial and poloidal basis functions to convert the quasi-neutrality integral equation into a matrix equation.

Recent extensions to the two-dimensional code have removed the restriction that the ion magnetic-drift frequency must be small compared to the mode-frequency,<sup>1,2</sup> and have improved the electron and ion collision operators to allow consideration of the plateau and Pfirsch-Schlüter regimes in addition to the banana regime. It is therefore possible to follow the transition from the trapped-electron mode to the well-known collisionless and collisional drift modes in toroidal geometry. In particular, an energy and pitch-angle dependent, number-conserving Krook operator is used for the electrons. Also the local code described in Ref. 3 has been extended to correspond as closely as possible to the present two-dimensional code. In general, the value of  $b_{i\theta} \equiv k_\theta^2 \rho_i^2 / 2$ , where  $k_\theta$  is the poloidal wavenumber, which gives the highest growth rate is slightly less than unity for small  $v_e^*$  and is smaller for large  $v_e^*$ , where  $v_e^*$  is the usual dimensionless electron collisionality parameter. Results for the variation of linear growth rates in minor radius, using model profiles for the variation of equilibrium quantities, will also be presented.

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\*Work supported by U.S.E.R.D.A. Contract E(11-1)-3073.

<sup>1</sup>G. Rewoldt, W. M. Tang, and E. A. Frieman, Phys. Fluids 20, 402 (1977)

<sup>2</sup>W. M. Tang, et al., Report PPPL-1279 (1976).

<sup>3</sup>J. C. Adam, W. M. Tang, and P. H. Rutherford, Phys. Fluids 19, 561 (1976).

THE MOTION OF IONS IN THE PRESENCE OF  
A TRAPPED-ION MODE OF FINITE AMPLITUDE \*

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ABSTRACT

A dissipative trapped-ion mode of moderate amplitude causes large perturbations in the motion of ions in a tokamak. Ignoring collisions, we study ion motion, both analytically and numerically, for a mode of amplitude  $\Phi_0$  given by  $e\Phi_0 = 0.05 \epsilon T_i$  and frequency  $\omega = \frac{1}{2} \omega_{bi}$ , where  $\epsilon$  is the inverse aspect ratio and  $\omega_{bi}$  is the bounce frequency of a deeply trapped, thermal ion.

The longitudinal action  $J$  is not even approximately invariant during the motion. Near the fundamental bounce resonance ( $\omega = \omega_b$ ) a new invariant, topologically different from  $J$ , appears. Near the trapped/circulating boundary in velocity space bounce resonances ( $\omega = n\omega_b$ ,  $n = 1, 2, 3, \dots$ ) can overlap each other, preventing the existence of any invariant associated with the parallel motion. The ion motion is then stochastic, even if the trapped-ion mode spectrum consists of only a single mode. We discuss possible saturation mechanisms for the trapped-ion mode as a result of these nonlinear effects in the ion motion.

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## ANOMALOUS TRANSPORT DUE TO DRIFT WAVES IN TOKAMAKS\*

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The existence of low frequency drift waves in tokamaks has already been demonstrated by experiments in the ATC and TFR devices. The effects that these electrostatic fluctuations have on the transport is investigated through moment equations of the drift kinetic equation in toroidal geometry. From the continuity equation, it is observed that the total particle flux is the sum of the neoclassical flux, the fluctuation  $\underline{E} \times \underline{B}$  flux, and a nonlinear version of the Ware pinch, which is usually smaller than the  $\underline{E} \times \underline{B}$  flux. From the energy moment equation, the heat flux can be expressed roughly as a sum of neoclassical and fluctuating  $\underline{E} \times \underline{B}$  heat fluxes. In addition, there are heat exchange terms, first pointed out by Krall and Liewer,<sup>1</sup> between electrons and ions due to the fluctuations. From the momentum equations, two results are obtained. First, there is an anomalous viscosity due to  $\underline{E} \times \underline{B}$  transfer of parallel momentum. Second, a bootstrap current appears in the Ohm's Law when the mode is not flute-like. For typical parameters, this bootstrap current is only about 15% of the total current. When impurities are taken into account, it is found that impurity diffusion due to fluctuating  $\underline{E} \times \underline{B}$  drifts is larger than, but about the same order of magnitude as the neoclassical flux. The direction of the fluctuating  $\underline{E} \times \underline{B}$  flux is usually outward so that the neoclassical prediction of impurity accumulation is difficult to realize in the presence of drift wave fluctuations.

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\*Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>1</sup>N. A. Krall and P. C. Liewer, Phys. Rev. Lett. 36, 1041 (1976).

Nonlinear Theory of Trapped Electron Turbulence\* A.K. Sen,

Columbia University. We consider the nonlinear turbulent spectrum of dissipative trapped electron instability driven by electron temperature gradient and finite Larmor radius and including the electron curvature drift resonances. It is found that  $\bar{E} \times \bar{B}$  turbulent diffusion and enhanced ion collisional damping are responsible for nonlinear saturation. With the rms fluctuation level thus determined, we develop a phenomenological theory of the shape of turbulent spectrum. It is shown that the spectrum is fundamentally dictated by the slightly non-equilibrium situation where energy dissipation in shear stabilized modes and the energy input at large wavelength unstable modes play critical roles. The resulting spectrum consists of a sub-spectrum of strong turbulence which sits between two sub-spectra of weak turbulence. In the long wavelength weak turbulence sub-spectrum decay interaction and energy input by unstable modes are important. In the short wavelength weak turbulence sub-spectrum nonlinear scattering by ions and energy dissipation in shear stabilized modes play dominant roles. In the intermediate wavelength strong turbulence sub-spectrum mode coupling due to polarization drift discussed by Mima and Hasegawa play the important role. The anomalous particle and thermal diffusion are determined with the help of the total spectrum.

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Mode Coupling Saturation of Drift Waves<sup>\*</sup>

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We consider numerically the saturation of collisionless drift waves through three wave resonant coupling in the presence of a sheared magnetic field. The principal energy sink is the nonlinear coupling of a mode generated near a rational surface to radially propagating modes which are quickly ion Landau damped. Estimates are given for the saturation level vs  $L_s/L_n$ .

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POSTER SESSION H

## Electron Cyclotron Heating in Tokamaks

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The recent development of multi-kilowatt millimeter wave sources at NRL and in the Soviet Union has motivated us to study electron cyclotron heating of tokamaks. Previous work has emphasized mode conversion of the extraordinary wave at the upper hybrid resonant surface<sup>1</sup> and subsequent absorption of the converted electrostatic wave.<sup>2</sup> In this scheme, it is difficult to control the position in the plasma at which the energy is deposited.

We examine the direct absorption of the ordinary and extra-ordinary mode at the first and second harmonic of the electron cyclotron frequency. We generally find good absorption; also use of this absorption mechanism allows one to select the point in the plasma at which the energy is deposited. Our calculations show that absorption efficiency of the ordinary wave is very high at the cyclotron frequency, but is negligibly small at the second cyclotron harmonic. On the other hand, the extra-ordinary wave is efficiently absorbed at the second cyclotron harmonic.

If there is a nonzero component of the wave vector parallel to the magnetic field, the wave can be launched from either the inside or outside of the torus. On the other hand, if  $k_{\parallel} = 0$ , most of the wave energy is reflected if the wave is launched from the outside of the torus. If it is launched from the inside, the wave is almost completely absorbed. Finally, the effect of anomalous scattering by parametric instabilities will be discussed.

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<sup>1</sup>V. E. Golant and A. D. Piliya, Sov. Phys. Usp. 14, 413 (1972)

<sup>2</sup>V. M. Kochetkov, Sov. Phys. Tech. Phys 21, 280 (1976)

Slow Wave Launching at the Lower Hybrid Frequency Using an Electrostatic Antenna.\* V. S. Chan and Charles Moeller, General Atomic Company--An alternate method to phased waveguide array for launching a slow wave at the lower hybrid frequency is by using an electrostatic antenna. By careful design, the antenna is capable of producing a well-defined parallel wavenumber spectrum that is suitable for lower hybrid electron heating experiments. The basic property that distinguishes the antenna from the waveguide array is the fact that it is only loosely coupled to the plasma. In other words, only a small fraction of the applied power is radiated by each element and the large fraction of reflected power can be carried by the transmission line along the antenna to be radiated by subsequent elements. Under the loose coupling assumption, the antenna can be analyzed using simple transmission line models. The plasma can assume various density profiles but coupling to the fast wave is neglected. The dependences of the coupling efficiency on different plasma parameters will be studied. The result of such study will be useful for the DIIA lower hybrid electron heating experiment.

\*Work supported by the U. S. Energy Research and Development Administration, Contract EY-76-C-03-0167, Project Agreement No. 38.

Cavity Eigenmode Heating of Tokamak Plasmas.\* Cheng Chu and

J. L. Sperling, General Atomic Company--The radio frequency wave heating of tokamak plasmas by cavity eigenmodes is analyzed. Using a computer code, MOTHRA, we are able to calculate both the field structures of the eigenmodes and wave heating patterns. Depending on the tokamak parameters, the cutoff frequency can vary from the intermediate range ( $\Omega_i < \omega < \Omega_e$ ) in small devices to a frequency well below the ion-cyclotron frequency ( $\omega < \Omega_i$ ) in power reactors. Generally speaking, for a given frequency, the shortest parallel wavelength modes have field structures concentrated deep within the plasma. Cavity eigenmodes with frequencies lying near the lower-hybrid frequency are predicted to exist in Doublet IIA. These modes which are actually whistler waves can be absorbed by linear electron and transit-time damping as well as by nonlinear processes. The application of cavity eigenmode heating, both below and in the ion-cyclotron range of frequencies, is analyzed for a doublet TNS. It is shown that traveling cavity eigenmodes can be used to drive the toroidal current in tokamak reactors.

\*Work supported by the U. S. Energy Research and Development Administration, Contract EY-76-C-03-0167, Project Agreement No. 38.

ANOMALOUS PLASMA HEATING BY ELECTROSTATIC WAVES  
DRIVEN UNSTABLE BY NEUTRAL BEAM INJECTION \*

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The fluctuating electrostatic field energy of waves driven unstable by energetic ion beam injection is studied as a possible source of plasma heating. Electrostatic ion beam-plasma modes are analyzed for both cold and warm ion beams propagating at an arbitrary angle with respect to the magnetic field. A new branch of the ion-acoustic mode is reported. The instability thresholds and growth rates are given for the modified two-stream, cross field ion-ion, ion-acoustic, beam-acoustic and lower-hybrid modes. The quasilinear moment equations are used to calculate the time rate of change of momentum and thermal energy of each plasma component in the presence of ion beam driven instabilities. The collisional slowing-down time of the ion beam is found to be typically more than an order of magnitude larger than the quasilinear slowing-down time for the most unstable modes.

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\*\* On leave from CONICYT, Buenos Aires, Argentina.

Stochastic Heating of Electrons in Spatially Localized RF Fields.\*

R. W. Harvey and J. L. Sperling, General Atomic Company--The RF electric field in the Doublet IIA lower hybrid heating experiment is expected to be greater than that for which quasilinear theory is valid. A Markovian model for the wave particle interaction is used to obtain a Fokker-Planck kinetic equation for the electrons. For sufficiently small field amplitudes, purely diffusive "quasilinear" behavior is exhibited. At the larger electric fields in the wave-particle trapping regime, we numerically integrate the single particle equations of motion to obtain the Fokker-Planck coefficients. Transport-like terms in velocity space are found to be important.

In order to check the range of validity of the above Fokker-Planck equation, we include Coulomb drag and diffusion directly in the single particle equations of motion. Results of numerical integration of these equations for an ensemble of particles are compared with the Fokker-Planck results in the regime where there is a quasisteady balance between RF and Coulomb collisional effects. The code provides estimates of the power absorption in the fully nonlinear regime, including effects of magnetic wells.

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## SIMULATION OF THE LHD INSTABILITY USING A 2-D HYBRID CODE\*

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A two-dimensional hybrid simulation code, TDHYD, has been developed for use in studying plasma phenomena on extended time and distance scales.<sup>1,2</sup> The model on which the code is based makes fundamental use of the small Debye length or quasi-neutrality assumption. The ions are modeled by particle-in-cell techniques while the electrons are considered a collision-dominated fluid. The fields are calculated in the non-radiative Darwin limit. The quasi-neutral counterpart of Poisson's equation in the hybrid model is obtained by summing the electron and ion momentum equations. Taking the divergence, the resulting equation can be put in the form

$$\nabla \cdot \mu \nabla = \chi$$

where  $\theta$  is the electrostatic potential and  $\mu$  and  $\chi$  are known functions of position.

One of the more striking features of the model is the retention of finite electron mass--permitting simulation of instabilities such as the LHD instability. As a first application of the code, a LHD unstable equilibrium with parameter accessible to both hybrid and full particle simulation has been selected. Initial results indicate quite good agreement between the two simulation methods in linear growth rate and wavenumber. Nonlinear phenomena are greatly influenced by the finite differencing techniques used in calculating the divergence of the electron kinetic energy tensor. Details of this and other comparisons will be presented.

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<sup>1</sup>D. W. Hewett, C. W. Nielson, and D. Winske, Proc. Meeting, Division of Plasma Physics, Paper 1F-13, San Francisco (1976).

<sup>2</sup>C. W. Nielson, D. Winske, and D. W. Hewett, Proc. Meeting on Theoretical Aspects of CTR, Paper 2A-17 (1976).

\*Work performed under the auspices of U.S.ERDA.

## Runaway Electrons in Collective Electric Fields

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The time development of a runaway distribution function is solved numerically using Fokker-Planck and self-consistent quasilinear diffusion equations. We find the anomalous-Doppler-shifted mode and the bump-on-tail mode can be excited whenever  $\omega_{pi} \leq \omega_k \leq \omega_{pe}$ . The frequency spectrum and the synchrotron radiation power resulting from such instabilities are consistent with experimental observation. In order to explain the long-time behavior of the synchrotron radiation experimental data, we find that there is an energy threshold, below which electrons are not significantly slowed down by wave scattering. This threshold increases in magnitude as the experiment progresses.

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<sup>\*</sup>Science Applications, Inc., McLean, Virginia

## MAGNETOACOUSTIC HEATING IN A SHARP-BOUNDARY PINCH\*

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In order to investigate the usefulness of magnetoacoustic effects for heating pinches, the Vlasov-fluid model (collisionless ions, massless fluid electrons) is being applied to the magnetoacoustic heating of a sharp-boundary screw pinch. The formulation of the sharp-boundary Vlasov-fluid model is that of Lewis and Turner.<sup>1</sup> The Liouville equation in the Vlasov-fluid model has been solved in the limit of small values compared to unity of  $kr_L$  and  $\omega/\omega_{ci}$ , where  $k$  is the largest relevant wavenumber and  $\omega_{ci}$  is the ion cyclotron frequency. Finite ion gyroradius effects are kept to first order, and the solution is valid up to the sharp boundary. This solution of the Liouville equation has been used to derive a differential equation for  $\underline{\xi}$  ( $\underline{A}^{(1)} = \underline{\xi} \times \underline{B}^{(0)}$ ) which contains finite gyroradius effects to first order, and the equation has been solved analytically. There appear to be interesting effects of finite ion gyroradius on magnetoacoustic heating, and numerical studies are proceeding.

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(1)H. R. Lewis and L. Turner, Nuclear Fusion 16, 993 (1976).

\*Work performed under the auspices of U.S.ERDA.

COMPUTER SIMULATIONS OF DRIFT ALFVEN WAVE INSTABILITIES\*

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A plasma confined in a magnetic field can suffer two types of low frequency instabilities, the usual drift wave and the drift Alfvén wave, which are driven by the plasma pressure gradient. In the limit of a homogeneous plasma these unstable waves go over to ion-acoustic and shear Alfvén waves that propagate along a direction almost perpendicular to the magnetic field. The drift Alfvén wave instability occurs when  $\beta$  is finite and the resulting magnetic field perturbation bends the magnetic field line. A newly developed two and one-half dimensional magnetostatic particle code is used to investigate this finite  $\beta$  instability. The frequency and growth rate of the unstable wave diagnosed at the place where the density gradient is maximum are quite close to those predicted by the local dispersion relation derived by Mikhailovskii and Rudakov.<sup>1</sup> For a finite but low  $\beta$  case the density flattening seems to limit the growth of the instability leading to saturation. A sheared magnetic field generated by the plasma current has been introduced into our model. The effects of a sheared magnetic field on the instability is currently being studied.

\*Work supported by ERDA.

<sup>1</sup>A. B. Mikhailovskii and L. I. Rudakov, Sov. Phys. JETP 621 (1963).

Monte Carlo Techniques for  
Neutral-Beam Injection Studies\*

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A clearer understanding of the effect of neutral-beam injection on the total system energy balance and the thermal/mechanical design of the first wall is essential for the development of mirror reactors. Consequently, a 3-D Monte Carlo (MC) computer code has been developed for the simulation of energetic neutral-beam injection into mirror devices. These calculations concentrate on the trapping of the beam and the angular and energy distributions of the charge-transfer neutrals that bombard the first wall.

The MC method employed is analogous to that originally developed and refined for neutron and gamma ray studies. A key and unique aspect of the present technique, however, is the incorporation of a generalized technique to rapidly evaluate the charge-transfer and ionization interaction rate coefficients at the interaction points. The MC approach is particularly appropriate as it can treat both the 3-D plasma geometry of a Yin-Yang system as well as the anisotropic plasma phase-space distribution. Collisional histories of a large number ( $3 \times 10^5$ ) of computer particles are tracked to give a statistical picture of the consequences of the competing charge-transfer and ionization collisions. A generalized sampling scheme allows the treatment of spatially non-uniform plasma density. The ionizational trapping and consequent depletion of the neutral beam is calculated at each collision point. Then, a charge transfer neutral is selected for further tracking. After several generations, a neutral may escape the plasma, hitting the first wall. The angular and energy distributions of the escaping neutrals are tabulated and used to calculate local heating and sputtering-induced erosion rates. Results are presented for injection into near-term concepts such as the Mirror Experiment (MX) and mirror FERF.

\* Work supported under the auspices of US ERDA.

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NONLOCAL CORRECTIONS TO POISSON'S EQUATION IN  
NONUNIFORM PLASMAS WITH FINITE GYRORADIUS\*

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ABSTRACT

In a magnetized plasma, Poisson's equation may be written in terms of a nonlocal susceptibility kernel  $\chi(\underline{x}, \underline{x}'; \omega)$ . This nonlocal form of Poisson's equation may be manipulated into an infinite-order differential equation, which may then be truncated at a convenient order in the parameter  $k_{\perp}\rho$  ( $\rho \equiv$  particle gyroradius), to obtain a tractable problem for studying the normal modes of the plasma. In this formulation,  $\chi$  may be expressed also as a power series in another parameter  $(\rho/L)$  ( $L \equiv$  scale length of plasma parameters); only the zero-order term of this series has been kept by previous authors. In plasmas of interest (e.g. the Tormac sheath), both  $k_{\perp}\rho$  and  $\rho/L$  may be comparable to unity, and so the higher-order terms in this expansion should be taken into account in calculating the frequencies and structure of the normal modes of such plasmas.

We use a slab model, allowing for arbitrary variation (in the  $x$ -direction) of the magnetic field magnitude and shear rate, and of the density of guiding centers. We obtain an explicit expansion of  $\chi$  to all orders in  $\rho/L$ . Each term of this expansion is a function of  $(k_{\perp}\rho)$ . The lowest order term has a form which reduces easily to the expressions for  $\chi$  usually employed in the limit of uniform magnetic field. Higher order terms in the expansion bear a simple relationship to ~~this~~ zero-order term.

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\* Work supported by U.S. ERDA, Division of Magnetic Fusion Energy.

# Anomalous Heating of Magnetically Confined Plasmas Near the Lower Hybrid Frequency\*

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Heating of magnetically confined plasmas via radio-frequency waves near the lower hybrid has been of current interest both in theory and laboratory experiments. While there is some experimental evidence that significant plasma heating can take place in the main body of a plasma, the detailed physical picture of wave propagation and plasma heating is not clear because of the limited accessibility for the plasma diagnostics.

Using the large scale, fully three-dimensional particle simulation models developed earlier, we have extended previous numerical simulations in one and two dimensions to full three dimensions where the plasma parameters for the simulations are comparable to those in laboratory plasmas.

For the case of uniform pump and a plasma, we observe three stages of instabilities. First, the parametric instability to low-frequency ion wave or quasimode takes place through the electrons  $c\tilde{E}_{ox} \times \tilde{B}_{oz}/B_o^2$  coupling which then drives secondary instability through  $c\tilde{E}_y \times \tilde{B}_o^2/B_o^2$  coupling. This process in general generates nearly isotropic two-dimensional turbulence in  $x - y$  plane elongated in  $z$  ( $k_x \sim k_y \gg k_z$ ). Finally, we observe the flute modes  $k_z \equiv 0$  are excited through the mode coupling process which eventually dominate the other modes.

Strong heating of both ions and electrons are observed as the excited electric field approaches the pump electric field. This heating is essentially due to quasilinear diffusion since there are so many waves that are excited simultaneously. When the instability saturates at  $E^2 \sim E_o^2$ , heating continues for a weak pump ( $E_o^2/4\pi nT_o \ll 1$ ) or becomes smaller for a strong pump ( $E_o^2/4\pi nT_o \sim 1$ ). A three-dimensional model which can treat the propagation of a pump in an inhomogeneous plasma will also be discussed.

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COLLISIONAL HEATING BY ALPHA PARTICLES AND THE TIME  
EVOLUTION OF THE ALPHA PARTICLE DISTRIBUTION IN FUSION REACTORS \*

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The collisional heating of the background electrons and ions by energetic alpha particles is studied using the Fokker-Planck equation. The time evolution of an initially spherically symmetric distribution of alphas is calculated and the time-asymptotic distribution is given for a constant source of alpha particles. The effect of a weak parallel electric field is included as a perturbation. The steady-state alpha particle distribution is rather flat for velocities much less than the initial velocity, but decreases rapidly for velocities near the initial velocity. An approximate asymptotic expression is given for the high energy tail of the alpha particle distribution. The moments of the alpha particle distribution are taken to determine the time rate of change of the density, momentum, energy and heat flux. The electron and background ion contributions to the drag and diffusion in velocity-space are calculated explicitly to show the effect of each species.

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A simple equation is derived describing the production of the quasi-steady magnetic field by resonant absorption in a cold, collisionless plasma. The method employed involves a direct integration of Ampere's law. The derivation exploits the fact that the generation of the magnetic field is unaffected by pressure and wavebreaking (or collisions) until one of these processes saturates the high frequency fields. Thus, the derivation is more transparent than those given previously in which either a stress-tensor<sup>1</sup> or Vlasov<sup>2</sup> formalism was employed, but the final results did not contain pressure or collisions explicitly. The source of the magnetic field is the beat current resulting from the nonlinear interaction of the incident light wave and the electron plasma oscillation. The magnetic field is localized to the skin depth about the critical surface. An initial value problem is solved to ascertain the explicit time behavior of the oscillating fields and a fully analytic expression for the quasi-steady magnetic field is presented. The amplitude and spatial variation of the magnetic field is in agreement with computer simulations.<sup>2</sup>

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\*Research supported by the Laser Fusion Feasibility Project.

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# CYCLOTRON HARMONIC ABSORPTION VIA THE GREEN'S FUNCTION FOR THE MODE CONVERSION-TUNNELING EQUATION

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## ABSTRACT

Cyclotron Harmonic absorption is included into the analysis of the mode conversion-tunneling process by solving for the Green's function of the fourth order mode conversion-tunneling (MCT) equation. For the first cyclotron harmonic where  $\omega \approx 2\omega_{ci}$  and the wave propagates exactly across the magnetic field, the solution is given in closed form in terms of the basic homogeneous solutions. For finite  $k_{\parallel}$ , the solution is given by solving an integral equation where the kernel is not self-adjoint as the basic MCT equation is not self-adjoint. For cavity modes, it appears that mode conversion losses are generally greater than harmonic cyclotron absorption losses although both are of the same order, and this result implies that electron heating may dominate ion heating for these processes.

SCATTERING OF CYCLOTRON RADIATION BY BERNSTEIN MODES<sup>\*</sup>

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In high temperature fusion reactors cyclotron radiation constitutes a serious power loss; serious enough to exclude low  $\beta$  systems, and a magnetically confined D-D fueled system. Since the primary radiation is in the infrared this loss can easily be overcome by using a metallic reflector around the reactor, thus establishing a thermal radiation field in the infrared. This is, of course, scattered into higher frequencies by the particles, but the process seems slow enough to be unimportant.

There is, however, a suspicious element. At the cyclotron harmonics not only the cyclotron radiation, but Bernstein modes are excited, and the possibility exists that resonance scattering may induce a much more rapid transfer of energy to high frequencies. This possibility will be explored!

<sup>\*</sup>Supported by NSF Grant PHY-75-02690.

COMPUTER SIMULATION OF SYNCHROTRON RADIATION  
IN A MAGNETIZED PLASMA\*

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The loss of kinetic energy of a plasma due to cyclotron radiation becomes very important when the electron temperature is high and the applied magnetic field increases. This energy loss may turn out to be a critical problem in the tokamaks, mirror devices, and bumpy torus. A  $2\frac{1}{2}$  dimensional relativistic electromagnetic code has been developed on UCLA's IBM 360/91 in order to study effects of plasmas on the radiation processes. Also a 2D formula of radiation power for current sources in circular motion has been derived, in both real space and Fourier space. This formula has been used to check the code for low density plasmas. The results agree with each other quite well. Synchrotron radiation processes in plasmas of energetic electrons have been observed in simulation and the result is checked by retaining higher order terms of  $V/C$  in the formula. Their damping times of energy loss are also derived and checked. The same model is coded and extended to a larger simulation system on the ILLIAC IV computer. The computation speed ratio of the ILLIAC IV to IBM 360/91 is about 40 and the cost effectiveness is a factor of 10 to 15.

\*Work supported by ERDA and EPRI.

LANGMUIR WAVES IN A SPATIALLY PERIODIC POTENTIAL\*

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An electron plasma with an immobile neutralizing background is immersed in a potential which is zero everywhere except on the parallel, equidistant planes  $x = 2nL$  ( $n$  integer). On these planes the potential assumes the value  $m_e v_M^2/2$ . Thus electrons with  $|v_x| > v_M$  are free while those with  $|v_x| < v_M$  are trapped and reflected between the planes. It is shown that this periodic potential introduces a damping of Langmuir waves which is in addition to Landau damping. This incremental damping, in terms of the thermal speed  $a = (2kT_e/m_e)^{1/2}$  is given by

$$\gamma = - \frac{\sqrt{2}a}{L} \left\{ 1 - \left( 1 + \frac{v_M^2}{a^2} \exp \left( - \frac{v_M^2}{a^2} \right) \right) \right\}$$

when the wave number  $k = 0$  (plasma oscillations) and tends to decrease as  $k$  increases. This damping can be understood as the result of the electrons being reflected by the potential barriers and thereby losing their momentum.

\*Work supported by ERDA and NSF.

†On leave from the Centre de Recherches en Physique des Plasmas, Ecole Polytechnique Fédérale, Laussane, Switzerland.

THE FLUID THEORY OF THE PONDEROMOTIVE EFFECT  
IN A MAGNETIZED PLASMA\*

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The fields  $\underline{E}$ ,  $\underline{B}$ ,  $n_\alpha$ ,  $\underline{v}_\alpha$  where  $\alpha$  numbers the species, are decomposed into slowly varying and rapidly varying components which satisfy "slow" and "fast" equations of motion respectively. The fast densities and velocities are expanded in powers of the fast electric field. The expansion coefficients contain terms which diverge in Fourier space at zero frequency. These terms must be transferred from the fast to the slow equations where they really belong. Thus the divergence is removed and the slow momentum and continuity equations are modified by the appearance of a force and a drift, respectively. This force and drift represent the generalized ponderomotive effect. The analysis is carried out to second order in the fast field. Two special cases are treated: monochromatic waves and spatially inhomogeneous turbulence.

\*Work supported by ERDA.

On leave from the Centre de Recherches en Physique des Plasmas, Ecole Polytechnique Federal, Laussane, Switzerland.

## PARTICLE SIMULATION STUDIES OF THE LOWER HYBRID DRIFT INSTABILITY\*

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The lower hybrid drift instability has been proposed as a mechanism for anomalous transport in both the implosion and post-implosion phases of high density pinches. Although the linear theory of this instability has been thoroughly investigated, the nonlinear theory--for example, the anomalous transport coefficients obtained from a quasilinear theory--is on less firm ground, since a detailed understanding of the saturation mechanism does not yet exist.

We present results of the nonlinear stage of the lower hybrid drift instability obtained from particle simulations. These two-dimensional calculations, which employ a non-radiative electromagnetic full particle code<sup>1</sup> and inhomogeneous sheath-like configurations,<sup>2</sup> have shown the existence of strong flute instabilities with properties of the lower hybrid drift instability.<sup>3</sup> Results shown here are a compilation of many cases, extending across a wide range of parameters. Scaling of the observed saturation level with plasma parameters and comparison with predictions of particular saturation mechanisms will be discussed.

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<sup>1</sup>C. W. Nielson and H. R. Lewis, Meth. Comp. Phys. 16, 367 (1976).

<sup>2</sup>D. W. Hewett, C. W. Nielson and D. Winske, Phys. Fluids 19, 443 (1976).

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\*Work performed under the auspices of U.S.ERDA.

R. PRICE, THEORY PROGRAM

## PLENARY SESSION PC

PC1, PC2

D. BAKER - TOROIDAL Z-PINCH  
(NO ABSTRACT)

MULTIPOLES AS REACTORS\*

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Some new ideas on how multipoles might be employed in reactors are looked at from a conceptual point of view. If advanced fuels which produce very low neutron fluxes can be made to work, then it appears possible to use properly shielded floating superconducting rings. In the event that such fuels can not be made to work, several alternatives appear. A supported multipole with current and cooling feeds could be operated like a super mirror with mirror losses occurring only on the diverted field lines which shield the leads. It is also possible to consider floating rings which will run hot and use the difference in heat flux on the two sides of the ring to generate a self sustained current. Finally it is possible to power the rings by means of neutral beams or possibly other energy sources; the rings would radiate away the ohmic dissipation plus the reactor power falling on them. General considerations of these ideas will be presented.

\*Work supported by ERDA.

# THE TANDEM MIRROR CONCEPT \*

B. Grant Logan \*\*

The physics principles of plasma confinement based on the tandem mirror concept <sup>1,2</sup> are reviewed, in which the ambipolar potentials of two conventional mirror plasmas are used to electrostatically plug the ends of a long solenoid. Two-dimensional Fokker-Planck calculations of ion lifetime  $\tau$  in a potential well with magnetic mirrors agrees closely with the analytic result of Pastukhov <sup>3</sup>:

$$\tau \approx \tau_{ii} g(R) \left(\frac{\phi}{T}\right) \exp\left(\frac{\phi}{T}\right), \quad (\phi \gtrsim 2T) \quad (1)$$

where  $\tau_{ii}$  is the Spitzer ion-ion collision time for  $Z=1$  ions, and where  $g(R)$  is a slowly varying function of the mirror ratio  $R$ . ( $g(10) \approx 3$ ). With plugging potentials determined by  $\phi \approx T_e \ln(n_{\text{plug}}/n_{\text{solenoid}})$ , an ion lifetime  $\tau \sim 100\tau_{ii}$  can be achieved with plugs denser than the solenoid by factors of 3 to 10, depending on the ratio of  $T_e/T_i$ . By making the volume of the main fusion plasma in the central solenoid sufficiently large compared to the end plugs, the overall system  $Q = (\text{central solenoid fusion power}) / (\text{central solenoid} + \text{mirror plug injection powers})$  can be made to exceed that of a conventional mirror reactor by an arbitrary factor.

Using  $n\tau$  given by eq.(1) in the solenoid, and a classical  $n\tau \approx 10^{10} E_{\text{inj}}^{3/2}$  in the mirror machine plugs, reactor studies show that an overall  $Q=5$  at a power level of 1000 MW(e) can be obtained with a 100-meter solenoid sustained solely by alpha particle heating and electron heating by the plugs with  $E_{\text{inj}} \approx 1\text{MeV}$ . Fusion power density is sufficient if beta  $\gtrsim .5$  in the solenoid. Removal of thermalized alphas by classical cross-field diffusion on the majority D-T ion species appears to be adequate for a solenoid radius  $\lesssim 20$ -40 ion gyro-radii. Areas requiring further investigation include: (1) MHD beta limits in the solenoid (ballooning); (2) beta limits in the plugs at a small mirror ratio (mirror mode); (3) microstability of plug boundaries (DCLC mode); (4) enhanced diffusion in solenoid due to density, temperature, and potential-gradient-driven instabilities (drift waves, Mikhailovsky modes). Parameters of a proposed tandem mirror experiment (TMX) at LLL will be described.

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INFLUENCE OF MAGNETIC SHEAR ON THE LOWER-HYBRID-DRIFT AND MIRROR-DRIFT-CONE  
INSTABILITIES WITH APPLICATION TO SHEATH STABILITY IN REVERSED-FIELD-PINCH  
(ZT) AND TOROIDAL BI-CUSP (TORMAC) CONFIGURATIONS<sup>†</sup>

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The toroidal bicuspid configuration (Tormac) has received considerable attention as a possible magnetic confinement configuration for fusion plasmas. The viability of this approach depends on maintaining a stable plasma-magnetic field interface with characteristic sheath thickness comparable to an ion Larmor radius ( $|L_N| \sim r_{Li}$ ). In this paper, we investigate the microstability properties of the Tormac sheath, with particular emphasis on the influence of strong inhomogeneities and magnetic shear on the lower-hybrid-drift instability (Maxwellian ions) and the mirror-drift-cone instability (loss-cone ions). It is assumed that  $T_e \ll T_i$ , and the influence of finite plasma beta (electromagnetic effects and  $\nabla B_0$  electron orbit modifications) is neglected in the present analysis. Moreover, emphasis is placed on high-frequency perturbations characterized by  $\omega_{ci} \ll |\omega_r + i\gamma| \sim (\omega_{ce} \omega_{ci})^{1/2} \equiv \omega_{LH}$  and  $r_{Li}^{-2} \ll k_{\perp}^2 \ll r_{Le}^{-2}$ . For modes with localization width  $\sigma \ll |L_N|$ , a detailed numerical study of the resulting dispersion relation has been carried out for a wide range of system parameters. To summarize the main results, for  $|L_N| \sim r_{Li}$ , it is found that magnetic shear can completely stabilize the lower-hybrid-drift instability, and lead to a reduction in growth rate of the mirror-drift-cone instability. The persistence of the mirror-drift-cone instability (at a reduced growth rate) in the presence of moderate magnetic shear suggests that the addition of a cold plasma component (as in the 2X-IIB mirror experiment) may be desirable to further reduce the growth rate and suppress any anomalous transport that may be associated with this instability. Finally, a detailed profile study of the influence of magnetic shear on the lower-hybrid-drift instability is presented for typical reversed-field-pinch (ZT) profiles.

<sup>†</sup>Research supported by Energy Research and Development Administration.

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Diffusion in Large-Aspect Toroidal Plasmas<sup>†</sup>

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Classical diffusion is examined for toroidal plasmas with large-aspect ratio and circular section for both high- $\beta$  (reversed-field pinch) and low- $\beta$  (tokamak) cases, following the Grad-Hogan theory for isothermal resistive diffusion. The present result includes heat conduction and derives the explicit form of the ordinary differential equations for the diffusion velocity.

In the high- $\beta$  case ( $\beta \sim 1$ ), it is shown that the toroidal effects are negligible and the normal velocity component is determined by the second order ordinary differential equation

$$[(\rho a^2 + B^2)(ru_r)'] - (B_p^2/r^2)' ru_r = I, \quad a^2 = \gamma p / \rho,$$

where the inhomogeneous term  $I$  involves heat conduction and resistive terms.

For the singular low- $\beta$  case, resultant ordinary differential equations for the diffusion velocity are also explicitly obtained and are valid for both the fast phase and the slow phase. The shifting of the flux surfaces are also examined together with the evolution of pressure, etc. For the slow phase, the relationship between the present results and the Pfirsch-Schlüter formulas is discussed.

In these calculations, the parallel heat conductivity is taken to be much larger than the other transport coefficients and the correspondent change in the formulae for toroidal and poloidal velocity components are also noted.

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# DRIFT WAVE TURBULENCE EFFECTS ON MAGNETIC STRUCTURE AND PLASMA TRANSPORT IN TOKAMAKS\*

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## ABSTRACT

We hypothesize a new mechanism by which drift wave type fluctuations, such as those observed in the ATC and TFR experiments, can saturate and cause anomalous electron heat transport in tokamaks. The model is based on the fact that since parallel (to the magnetic field) transport processes are many orders of magnitude more rapid than perpendicular ones, very small helically-resonant magnetic perturbations (e.g.,  $\tilde{B}_r/B \gtrsim \sqrt{\chi_\perp/\chi_\parallel} \sim 10^{-4}$ ) that cause field lines to move radially can allow the parallel transport processes to contribute to radial electron heat transport. We propose that the magnetic perturbations<sup>1</sup> driven by the mostly electrostatic drift wave turbulence that is observed experimentally cause such effects in present tokamaks.

Small, resonant magnetic perturbations can easily tilt the magnetic field lines and modify the flux surface topology in a tokamak. The magnetic island structures produced by the magnetic perturbations<sup>1</sup> due to drift waves at any nonzero plasma  $\beta$  are studied<sup>2</sup> through the use of a helical flux function. For  $e\tilde{\phi}/T_e \sim 10^{-2}$ , the maximum  $\tilde{B}/B$  is about  $10^{-3}$ ; the width ( $\delta \sim 0.5$  cm) of the induced magnetic island structures is found to be slightly greater than the ion gyroradius. In a drift wave oscillation period, electrons circumnavigate a magnetic island whereas the slower moving ions see only a tilt of the magnetic field lines. Thus, electrons try to diffuse radially more rapidly than ions, but a radial potential builds up on the very short bounce time scale to keep the particle diffusion ambipolar. However, this rapid electron diffusion process does cause the electron temperature profile within the island to be flattened. The net radial electron heat conduction coefficient produced by an ensemble of adjacent island structures is estimated to be  $\chi_e \sim (\nu + \gamma)\delta^2$ , where  $\nu$  is the electron-ion collision frequency and  $\gamma$  is the drift wave growth rate. Other effects on transport processes will also be discussed. The maximum drift wave fluctuation level can be determined in this model from an estimate of the free energy available to a given magnetic island structure.

\*Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>1</sup>W.M. Tang, C. S. Liu, M. N. Rosenbluth, P. J. Catto and J. D. Callen, Nuclear Fusion 16, 191 (1976).

<sup>2</sup>J. D. Callen, G. G. Kelley, and B. V. Waddell, "Magnetic Island Characteristics in Tokamaks" (to be published).

<sup>3</sup>J. D. Callen, "Possible Effects of Drift-Wave Turbulence on Magnetic Structure and Plasma Transport in Tokamaks" (to be published).

## QUASILINEAR SIMULATION OF MIRROR MACHINES\*

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## ABSTRACT

We report here the progress in the simulation of transport processes in mirror machines using the quasilinear code. The quasilinear equations have been improved by solving for the wave number spectrum and the growth rates at the cyclotron harmonics using the local dispersion relation.

An interesting phenomenon that arises is the presence of r.f. "bursts". We show that under appropriate conditions, the steady state quasilinear diffusion level is unstable. Hence, instead of a constant level of diffusion, the diffusion coefficient, which is proportional to the r.f. energy, oscillates in time, in qualitative agreement with experiment.

We have attempted to assess the effect of radial gradients by including fixed gradients in the equations. Under these circumstances, quasilinear theory can only stabilize the mode if a radial temperature gradient as well as a density gradient is included. A 2-D quasilinear code, that includes radial gradients self-consistently, is currently being developed and preliminary results show that a radial ion temperature gradient is indeed induced. With the radial gradients, we find that, as in experiment, the first harmonic is a dominant part of the r.f. spectrum, even when higher harmonics can be potentially excited.

Using a high energy deuterium beam injection, we have observed the sensitivity of the plasma steady state to low energy deuterium and hydrogen stream. As in experiment, we find the steady state plasma is insensitive to stream species.

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ANOMALOUS TRANSPORT OF ALPHA-PARTICLES IN TOROIDAL FUSION  
PLASMA DUE TO SHEAR ALFVEN WAVE\*

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Fusion-born trapped  $\alpha$ -particles can interact with the shear Alfven wave ( $\omega = k_{\parallel} V_A \approx p\omega_b$ ,  $k_{\perp} \delta r \sim 0(1)$ , where  $V_A$  is Alfven speed,  $p\omega_b$  is a multiple of the  $\alpha$ -bounce frequency and  $\delta r$  is the  $\alpha$ -banana width) and induce instability.<sup>1</sup> We show that the ensuing turbulence causes substantial particle loss, energy loss and  $\alpha$ -density profile spreading. A theoretical model using quasilinear theory to describe the evolution of the  $\alpha$ -distribution function and the unstable spectrum is derived. In particular, the inclusion of a turbulence generated loss term  $-\nu_L f_{\alpha}$  in the kinetic equation can delay or even prevent the attainment of  $\partial f_{\alpha}/\partial E < 0$ . Here,  $\nu_L = D_{xx}/d^2(t)$ , where  $D_{xx} = \sum_p \sum_k (c^2/2B^2) |E_k|^2 \delta(\omega - p\omega_b) J_p^2(k_{\perp} \delta r)$  and  $d(t)$  is the  $\alpha$ -density profile width, determined self-consistently from  $\partial n_{\alpha}/\partial t - \partial/\partial x (\bar{D}_{xx} \partial/\partial x n_{\alpha}) = S_{\alpha}$ . A computer code that includes turbulent and collisional effects has been developed. Under typical reactor conditions ( $a/R \sim 1/4$ ,  $q \sim 3$ ,  $R \sim 500$  cm,  $B \sim 30$  kG,  $T \sim 20$  keV,  $n_e = 2 n_d = 2 n_t = 10^{14}$  cm<sup>-3</sup>) more than 50% of the trapped  $\alpha$ -particles are lost and the  $\alpha$ -density profile width doubles within roughly one slowing down time. The energy loss can be slightly less than the particle loss since a small part of the  $\alpha$ -particle energy is transferred to the background plasma before the  $\alpha$ 's are lost. The turbulence sustains itself for more than one slowing down time and the  $\alpha$ -distribution shows nonclassical features that may include other instabilities. These deleterious effects scale up with increasing  $a/R$ ,  $q$  and  $T$ , but scale inversely with magnetic field strength.

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1. T. D. Kaladze, A. B. Mikhailovskii, Kurchatov Institute Report IAE 2679.

## A Resonance Effect for Impurity Ions in Tokamak Plasmas\*

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## Abstract

It has been found that the lowest order flow of impurity ions on a magnetic surface undergoes a resonance type behavior at a critical value of the radial electric field ( $E_r$ ). The poloidal electric drift of the impurity ions ( $V_{z\theta} \approx -E_r/B$ ) has an effective phase velocity along the magnetic lines of force equal to  $-E_r/B_\theta$ . At values of the phase velocity  $-E_r/B_\theta$  which are supersonic for the impurity ions, the density variation  $\tilde{n}_z$  caused by the poloidal electric field ( $-\partial\Phi/r\partial\theta$ ) is  $\pi$  out of phase in  $\theta$  with the usual density variation given by the Boltzmann factor ( $-e\Phi/T$ ). At a critical magnitude of  $E_r$  (the resonant value) this part of  $\tilde{n}_z$  cancels the Boltzmann factor terms of the electrons and hydrogen ions and  $e\tilde{\Phi}(\theta)/T_i$  becomes very large, namely of order  $(\omega_{bi}/\nu_i)(r/R)$  if  $Z\bar{n}_z \sim \bar{n}_i$ , where  $\omega_{bi}$  and  $\nu_i$  are the hydrogen ion bounce and collision frequencies. Electrostatic trapping is then substantially more important than magnetic trapping since  $\omega_{bi}/\nu_i \gg 1$  for hydrogen ions in the plateau regime. For  $Z\bar{n}_z \sim \bar{n}_i$ , the resonance is broad in  $E_r$  and the values of  $E_r$  predicted by the neo-classical theory for present tokamak plasmas lie within this range. For lower impurity concentrations, the resonant peak in  $e\tilde{\Phi}(\theta)/T$  is both lower and narrower, but in Alcator electrostatic trapping could still be of comparable importance to magnetic trapping at the resonance. The effects of this resonance on neoclassical transport and the neoclassical disruptive instabilities will be discussed.

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POSTER SESSION I

RECENT RESULTS IN MICROWAVE HEATING THEORY FOR EBT<sup>\*</sup>

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The ray tracing code used to study microwave heating in EBT has recently been improved so as to permit operation with 3-D mirror equilibria and to include wave damping due to finite temperature effects. The rays are obtained by integrating the Hamiltonian form of the geometrical optics equations using the Appleton-Hartree dispersion relation. The wave damping is determined by integrating the imaginary part of  $k$  as obtained from warm plasma theory along the rays. It is found that in a mirror plasma having parameters similar to those projected for EBT-II ( $n_e \gtrsim 5 \times 10^{13}/\text{cm}^3$ ,  $T_e \gtrsim 2.5 \text{ keV}$ ,  $\omega_\mu = 120 \text{ GHz}$ ), a significant amount of heating is produced by the ordinary mode. The ordinary mode is not encumbered by questions of accessibility due to the right-hand cutoff as is the extraordinary mode. Also, calculations of cyclotron wave tunneling through the right-hand cutoff to the cyclotron resonance with finite dissipation included indicate much greater energy absorption than is obtained in Budden tunneling with no dissipation.

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\* Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

# A PRELIMINARY LOOK AT POSSIBLE TRAPPED PARTICLE MODES IN EBT<sup>\*</sup>

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## ABSTRACT

Because of the potential difficulty posed to tokamaks by the dissipative trapped-ion mode, it is of some interest to consider the possible role which trapped-particle modes might play in EBT. However, since EBT is fundamentally different from tokamaks, the approximations used in obtaining growth rates and stability boundaries for tokamaks are not applicable for EBT. In particular, EBT has no rotational transform, the population of trapped particles is larger than the population of passing particles, the magnetic drift frequency is comparable to the diamagnetic frequency ( $\omega_D \sim \omega_*$ ) and the collisionality is quite low ( $\nu_{ei} \leq \omega_{*min}$ ). In several respects, trapped and passing particles exchange roles in going from tokamaks to EBT. We have derived a dispersion relation for EBT using a very simple slab drift kinetic model much in the spirit of early work on trapped-particle modes in tokamaks. We find two new modes, a  $k_{||} = 0$  passing-particle mode and a trapped-particle mode with finite  $k_{||}$ . Because of the low collisionality and large  $\omega_D$ , both modes are essentially interchange modes and as such they are only likely to be dangerous in the C-mode of operation. In the desirable T-mode of operation they are stabilized over almost the entire profile of a typical EBT plasma. We feel the effect of such instability is a slight modification of the profile to bring the plasma to marginal stability.

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<sup>\*</sup>Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

COMPUTER SIMULATION OF FIELD REVERSED MIRROR  
CONFIGURATIONS WITH THE SUPERLAYER CODE\*

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ABSTRACT

We present the newest results from the Superlayer code on simulation of field reversed mirror configurations. A Monte Carlo prescription for modeling quasilinear effects of r.f. turbulence (specifically the drift loss cone mode)<sup>1</sup> has been implemented. We show comparisons of steady states achieved with and without the r.f. Plasma buildup (and some heating) occur in the presence of a certain low level of turbulence, in agreement with results of Ref. 1.

Some results for large systems,  $L/\rho_i \gg 1$  will be shown.

Predictions from recently completed proper deposition algorithm will be compared to our older results where we assumed all of the beam current to be trapped. We also show results from new diagnostics that can be more directly compared to the 2XIIB experiment: line density profiles and neutral analyzer scans. The latter resolves the neutral flux emerging from the plasma into various angles and into energy bins; in the experiment it is the primary technique for determining the energy distribution of the plasma.

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\*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

<sup>1</sup>H. L. Berk and J. J. Stewart, "Quasilinear Transport Model for Mirror Machines", UCRL-78407, to be published in Phys. Fluids

## TANDEM MIRROR CONFINEMENT STUDIES\*

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## ABSTRACT

The tandem mirror confinement concept has been studied using several different computational models: (1) coupled rate equations for each plasma species (plug ions, central cell ions including alpha particles); (2) coupled one-dimensional Fokker-Planck equations for ions plus rate equations for electrons (3) coupled two-dimensional Fokker-Planck equations for ions plus rate equations for electrons. The models have been compared for steady-state and some time-dependent problems, and models (1) and (3) have been found to be in good agreement. The coupled rate equation package has been used to study startup modes and schemes for removing thermalized alpha particles in tandem mirror reactors. A modified rate equation model, in which the plug parameters are restricted by the requirement of drift-cyclotron loss cone stability, has been used to study the operation of the planned TMX experiment at Livermore.

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\*Work performed under the auspices of the U. S. Energy Research and Development Administration under contract No. W-7405-ENG-48.

# Numerical Study of the Stability of Field-Reversed Ion Rings\*

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The generalized "energy principle" of Sudan and Rosenbluth<sup>1</sup> is used to investigate the low frequency ( $\omega < \omega_{ci}$ ) stability of field reversed ion rings in a background plasma. A numerical code has been written which determines sufficient conditions for stability by minimizing the free energy with respect to plasma displacements orthogonal to magnetic field lines  $\xi_{\perp}$ . Quasineutrality is guaranteed by a constraint which relates  $\xi_{\perp}$  to the beam perturbation. The displacement  $\xi_{\perp}$  is represented by a finite element expansion perpendicular to field lines and by a Fourier series along field lines. The Fourier modes couple strongly in the region of open field lines. Results will be presented for the case of a high density beam.

<sup>1</sup>Sudan, R. N. and Rosenbluth, Phys. Rev. Lett. 36, 972 (1976).

\*Work supported by U.S. E.R.D.A. Contract # EY-76-S-02-3170.

A LINEARIZED THREE DIMENSIONAL CODE FOR THE STUDY OF  
STRONG ION RING STABILITY\*

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J. Denavit - Northwestern University

To further investigate the stability of strong ion rings, we are developing a particle code which models low-frequency nonaxisymmetric perturbations, of given toroidal mode number  $l$ , about an axisymmetric zero-order state. This state may be taken to be an equilibrium, or, since zero-order fields are calculated self-consistently, a time varying axisymmetric configuration such as a ring undergoing "breathing" motion.

A field quantity  $\underline{A}$  is represented by:

$$\underline{A}(r, \phi, z) = \underline{A}^0(r, z) + \underline{A}^C(r, z) \cos(l\phi) + \underline{A}^S(r, z) \sin(l\phi),$$

where  $|\underline{A}^C|, |\underline{A}^S| \ll |\underline{A}^0|$ . Similarly, simulation particles given by:

$$\underline{x} = \underline{x}^0 + \underline{\delta x}^C \cos(l\phi) + \underline{\delta x}^S \sin(l\phi),$$

where  $\underline{x}^0$  gives the zero-order location of the particle in the  $r - z$  plane and  $\underline{\delta x}^C, \underline{\delta x}^S$  are the components of the small displacements associated with the perturbation. The latter may contain a nonzero azimuthal part, corresponding to a periodic bunching of the points comprising one simulation particle. Coupling of modes of different  $l$  is ruled out by the linearization formalism (1).

At present, a simple magnetostatic plasma is in use, assuming charge neutrality but no current neutralization. Zero-order radial and axial currents are neglected, since the ring is close to equilibrium. Thus, the field equations, solved by S.O.R., are:

$$\nabla x \nabla x \underline{A}_\phi^0 \hat{\phi} = J^0 \hat{\phi}, \quad \nabla x \nabla x \underline{A}^{C,S}, \quad \underline{E} = - \partial \underline{A} / \partial t$$

More realistic plasma models are under development.

Particles are advanced in rectangular coordinates(2). Predictor-corrector methods are used for both zero- and first-order. The zero-order predictor takes advantage of the approximate conservation of  $P_\phi$  for each particle (3). Preliminary results, using only 240 particles, will be presented.

- (1) J. A. Byers, Proc. 4th Conf. on Numerical Sim. of Plasmas.
- (2) J. Boris, Proc. 4th Conf. on Numerical Sim. of Plasmas.
- (3) A. Friedman, R. L. Ferch, R. N. Sudan, and A. T. Drobot, Cornell Univ LPS Report Number 190 (to appear in Plasma Physics).

\*Work supported by U.S. E.R.D.A. Contracts EY-76-S-02.2200.\*000 and EY-76-S-02-3170.

End Loss Reduction in Linear Devices  
by Means of Oscillating Multiple Mirrors<sup>†</sup>

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We present the results of an initial investigation of the effects of adding time-oscillating multiple mirror magnetic fields to a linear solenoidal plasma column. Previous studies<sup>(1)</sup> have shown rather dramatically that static multiple mirror fields can reduce the particle end loss by substantial amounts. However, the resulting bumpy linear column is very unstable to a long wavelength  $m = 1$  mode according to linearized MHD theory. Borrowing results obtained by Berge<sup>(2)</sup>, we show that it should be easy to dynamically stabilize the unstable bumpy column at frequencies and field modulations which are experimentally feasible. We show further that the rf energy in the dynamic stabilization circuit can be used for plasma heating since the oscillating field components give rise to strongly Landau damped ion-acoustic waves.

(1) Logan, B.G., et. al., Phys. Fluids 17 (1974) 302

(2) Berge, G., Phys. Fluids 13 (1970) 1031.

<sup>\*</sup> Permanent address - Institute for Plasma Physics, Nagoya University, Nagoya, Japan

<sup>†</sup> Work supported by U.S. ERDA Contract EY-76-C-02-3077.

RADIAL TRANSPORT WITH A SELF-CONSISTENT ELECTRIC FIELD  
IN THE ELMO BUMPY TORUS<sup>\*</sup>

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ABSTRACT

Neutral and charged particle densities and temperatures with a self-consistent ambipolar electric field are calculated as functions of radius for the toroidal core plasma in EBT I and EBT II. Effects of the hot electron annulus are neglected along with loss regions where gradient B and electric field drifts cancel. Moments of the bounce averaged drift kinetic equation are followed in time to equilibrium using ambipolar diffusion rates based on classical and neoclassical results with enhanced losses at low collisionalities. The electric field is calculated from quasi-charge neutrality while the neutral density and temperature are calculated using the Oak Ridge neutral transport model. Results are compared to an ad hoc electric field model used previously in both 0-dimensional and 1-dimensional models. Effects of variations in discharge conditions such as microwave power and neutral density and energy are considered.

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\* Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

Effect of plasma rotation and Hall current  
on linear tearing modes in reverse field pinches <sup>†</sup>

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Reverse field theta pinch experiments<sup>(1,2)</sup> have shown that the resistive tearing mode is not always observed. The most striking cases were discovered by Eberhagen<sup>(1)</sup> in experiments with a rather long thin  $\theta$ -pinch column with trapped reversed field; stability against tearing modes was noticed for times as much as an order of magnitude longer than the classical tearing mode growth rate. Strong plasma rotation was observed with the plasma column eventually breaking up due to rotating  $m = 2$  instabilities. A numerical calculation of  $m = 0$ ,  $k_{||} \neq 0$  linear resistive tearing modes for this geometry was made by Kaleck<sup>(3)</sup>; the effects of plasma rotation and Hall current in the Ohm's law were included. Kaleck found stabilizing influences of rotation and Hall effect and a qualitative scaling of growth rate which was consistent with experiment. We have begun an analytical investigation of  $m = 0$ ,  $k_{||} \neq 0$  tearing modes in reverse field geometry including plasma rotation and Hall effect. The linearized MHD equations are solved in two regions, the resistive boundary layer surrounding the singular surface, and the outer more MHD like region. Matching asymptotic solutions yields a dispersion relationship for the growth rate as a function of the Hall parameter, rotation frequency and geometric features of the plasma such as sheath width and compression ratio. The results of this work should have importance to planned liner experiments where reverse field  $\theta$ -pinch columns will be used as the plasma source.

(1) Eberhagen, A., Grossmann, W., Z. Physik 248, 130-149 (1971)

(2) Becker, G., et. al., in Proceedings of the 5th Conference on Plasma Physics and Controlled Nuclear Fusion Research, Tokyo (1974) paper IAEA-CN-33/E-4-1

(3) Kaleck, A., personal communication

\* Permanent address - New Jersey Institute of Technology, Newark, N.J.

<sup>†</sup> Work supported by U.S. ERDA Contract EY-76-C-02-3077.

# Ring-Plasma Electrostatic Instability; Linear Theory and Classification of Regimes; Simulation, Verification and Nonlinear Behavior

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A neutral beam injected normal to  $\underline{B}_0$  into a target plasma becomes a charged ring in  $v_\perp$  space and sets up an ion-ion velocity space instability, which has been given by many authors. We have examined this magnetized ring-plasma in some detail, for  $(\omega_p/\omega_c)^2 \approx 1000$  for the target plasma ions, with (like 100 keV D ring, 1 keV T plasma),  $v_{\text{ring}}/v_{\text{t plasma}} \approx 13$  and  $R \equiv n_{\text{ring}}/n_{\text{plasma}}$  varying from 0.03% to 200%. We have considered mainly the ion-ion interactions, as adding the electrons only shifts the frequencies a little.  $\underline{k}$  is perpendicular to  $\underline{B}_0$  and the electrostatic approximation, zero  $\beta$ , is used.

Small amplitude theory shows that there are at least three regimes. For a weak ring,  $0.03\% \leq R \leq 0.5\%$ , we find that the maximum growth rate is  $\gamma/\omega_{ci} \sim R^{0.6}$ ; for some of this regime the ring-plasma instability is due to coupling between the ring Bernstein ion-cyclotron harmonics and the plasma lower hybrid wave, and is like that observed in the laboratory by Seiler, et al.<sup>1</sup> In the intermediate ring case, with  $0.5\% \leq R \leq 5\%$ , the growth rate goes as  $R^{0.35}$ . For a strong ring, with  $5\% \leq R \leq 100\%$ , the growth rate varies as  $R^{0.17}$ ; here the coupling is between ring ion cyclotron waves and an ion acoustic-type mode excited by the ring-plasma combination, similar to the one seen in the laboratory, by Bohmer<sup>2</sup>. We have made simulations over the above ranges of  $R$  and observed the predicted linear behavior remarkably closely, using a linearized fluid model for the plasma and (fully nonlinear) particles for the ring.

The nonlinear behavior observed in the simulations show an average slowing and a broadening of the ring in  $v_\perp$  space, on the scale of the ion cyclotron time. Considerable detail has been seen in  $v_\perp$  space, occasionally with radial limits imposed by zeros of Bessel functions along the lines proposed by Aamodt.<sup>3</sup>

The zero  $\beta$  approximation will be discussed, along with steps to remove it.

1. Seiler, Yamada, Ikezi: P.R.L. 37, 700 (1976).
2. Bohmer, H.: P.F. 19, 1371 (1976).
3. Aamodt, R.E.: P.F. 13, 2147 (1970).

THERMAL STABILITY OF EBT OPERATING POINTS  
FROM A NEOCLASSICAL POINT MODEL\*

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Recently a neoclassical point model of EBT power balance has been formulated which is in rough agreement with experimental observation.<sup>1</sup> One feature of the model is that there are two neighboring equilibrium solutions, one with positive and one with negative ambipolar electric field. These two equilibria scale differently with density. This paper describes a routine to test the thermal stability of the various states. The results of the calculations show that it is possible to distinguish between the operating points on the basis of linear stability. For normal pressure profiles in the bulk plasma the stable solution corresponds to positive radial electric field while for bulk pressure peaked off center, the stable solution corresponds to negative electric field. These stability calculations then provide a further requirement, in addition to power balance, that the experiment must satisfy for the validity of a neoclassical description.

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\* Work supported by the U. S. Energy Research and Development Administration.

1. C. L. Hedrick, E. F. Jaeper, D. A. Spong, G. E. Guest, N. A. Krall, J. B. McBride and G. W. Stuart, Report #LAPS-23/SAI-77-566-LJ (submitted to Nuclear Fusion).

# Rotational Damping in Tormac<sup>\*</sup>

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In the tormac device, the plasma pressure is held by a thin mirror confined sheath supported by an external poloidal cusp magnetic field. In one version, the bulk of the plasma is penetrated only by toroidal magnetic field lines and the resulting toroidal drift could propagate the sheath loss-cone distribution throughout the plasma thereby reducing the overall confinement time to that of a mirror device. However, this propagation will not in general result in an ambipolar charge loss, so a self-consistent internal electric field arises. This sets the interior plasma in rotation and closes the particle drift orbits thereby halting the inward propagation of the sheath loss-cone distribution. To evaluate the magnitude of this effect, we have calculated the damping rate of plasma poloidal rotation in a torus with pure toroidal field lines. The results can be given in terms of the return current which acts to decrease the ambipolar electric field. The damping process was evaluated by using a modified form<sup>1</sup> of the CGL equations to include a relaxation time between the parallel and perpendicular pressures (for each species). We find agreement among three different ways of calculating the return current: using the momentum equations for each fluid species, using energy conservation and the rate of entropy increase, and using  $J_{\perp} = \sigma_{\perp} E_{\perp}$  with the orthogonal conductivity estimated from effective mean-free paths.

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<sup>\*</sup>Work supported by Electric Power Research Institute, Inc.

<sup>1</sup>A. H. Boozer, PPL Rept. EPRI-4 (1976).

A STEADY STATE GUIDING CENTER MODEL  
FOR THE COUNTERSTREAMING ION TORUS (CIT)\*

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In the CIT reactor (Jassby, D.L., Nucl. Fusion, 16 (1976), 15) counterstreaming ion beams are created by impact ionization and charge exchange of tangentially injected neutral beams. The net toroidal current ( $I_b \sim 1$  Mamp) in the system is driven by the ion beams ( $E_b \sim 80$  to 120 keV) which move in a thermalized electron background. At steady state there is a balance between reinjection of energetic ions and particle losses due either to drift to the limiter or the removal of cold ions which have been slowed down by electron drag.

Steady state solutions for the CIT are found using a self consistent guiding center model. The electron drag term has been added to the formal two time guiding center expansion and appropriate modifications are made to the guiding center model. The evolution to steady state for a typical set of CIT parameters will be presented.

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\*Work performed under the auspices of the U.S. Energy Research and Development Administration Contract No. W-7405-Eng-48.

Reduction in Residual Aberration Effects on  
Ion Optics in Neutral Beam Generators\*

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Solution of the Poisson-Vlasov equations for ions extracted from a plasma are obtained over a wide range of circumstances of interest. Equilibrium plasma electrons are included and the solution extends far back into the source plasma so that the plasma sheath position and width is part of the solution. In particular four types of ion accelerators have been studied, three of which yield better beam convergence than the conventional cylindrical bore triode. These three types of accelerators are: (a) a cylindrical bore tetrode, (b) a triode with various shapes of the plasma electrode, and (c) a triode with the plasma electrode at elevated potential from the source plasma maintained by use of an insulator. All of these designs reduce the residual aberration fields from that of the cylindrical bore triode for reasons which are different for each type. We will explain these reasons as well as the virtues and potential difficulties with each of these types, by reference to the actual transmission of ions compared to those available, the structure of the ion beam velocity distribution function and stability with respect to source plasma fluctuations.

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\*Research sponsored by the Energy Research and Development Administration under contract with the Union Carbide Corporation.

ACCELERATION OF A MAGNETICALLY INSULATED  
FOIL TO ULTRAHIGH VELOCITIES

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It is shown that a thin foil, magnetically insulated by a strong externally applied axial magnetic field, can be electrostatically accelerated by a magnetically confined electron cloud, produced by a relativistic electron beam generator, to velocities up to  $\sim 10^9$  cm/sec. With a velocity of  $\sim 10^8$  cm/sec, attainable by the acceleration over a distance of several meters and by magnetically compressing the foil after its acceleration into a compact projectile, conditions for the ignition of thermonuclear microexplosions can be reached by directing the projectile onto a suitable target. In comparison to other electron beam microexplosion concepts the required electron beam currents are here much lower and of the order  $10^3 - 10^4$  A due to the relatively slow energy cumulation in the foil.

COMMENTS ON GUIDING CENTER MOTION  
IN HIGH  $\beta$  TOKAMAKS<sup>\*</sup>

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ABSTRACT

In high- $\beta$  tokamaks, the guiding center orbits derived using the constants of motion ( $\mu$ ,  $\varepsilon$ ,  $P_\phi$ ) are not the same as those derived directly from the guiding center equations unless  $\nabla \times \bar{B} \approx 0$  (low  $\beta$ ). Because the difference in the two approaches is second order in gyroradius, we have looked at other higher order terms in the acceleration and velocity of the guiding center.

In doing this, we have encountered misconceptions and errors in the literature, particularly in connection with the parallel velocity and acceleration. We have found that some guiding center equations maintain their form when extended to higher order. In these cases, all that is necessary is to include the higher order terms in the magnetic moment.

Conservation of toroidal canonical angular momentum for the guiding center,  $P_\phi$ , holds through first order. Unfortunately, when second order drifts are included, the usual definition of  $P_\phi$  does not lead to a conserved quantity.

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\* Research sponsored by Energy Research and Development Administration under contract with Union Carbide Corporation.

Neoclassical Transport Coefficients for the  
ELMO Bumpy Torus Device\*

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Estimates of neoclassical transport rates for energy and particles are an important ingredient in modeling the present EBT experiment as well as in larger future devices. Particle transport in non-axisymmetric devices such as the bumpy torus occurs as the result of like-particle collisions and loss rates for electrons and ions are not automatically equal. In order to maintain quasineutrality, an electric field develops which modifies transport rates both through its direct influence on the cross-field mobility and by its effect on the centering of the orbits (thus changing the basic step size and transport coefficients). Transport coefficients and mobilities have been calculated using a bounce-averaged drift kinetic equation and making local approximations to density and temperature gradients. These exhibit a strong dependence on the parameters  $e\phi/kT$  and  $\nu/\Omega$  ( $\phi$  = radial potential,  $\nu$  = collision frequency,  $\Omega$  = drift frequency). Several different models for the collision operator have been used, including: particle and energy conserving Krook operator, Fokker-Planck operator with constant coefficients, and the Landau operator. Results of lifetimes and transport coefficients will be presented for the EBT-I and EBT-II devices.

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\* Research sponsored by the Energy Research and Development Administration under contract with Union Carbide Corporation.

<sup>†</sup> consultant from the University of Tennessee.

Reduction in Residual Aberration Effects on  
Ion Optics in Neutral Beam Generators\*

J. H. Whealton  
Oak Ridge National Laboratory

Solution of the Poisson-Vlasov equations for ions extracted from a plasma are obtained over a wide range of circumstances of interest. Equilibrium plasma electrons are included and the solution extends far back into the source plasma so that the plasma sheath position and width is part of the solution. In particular four types of ion accelerators have been studied, three of which yield better beam convergence than the conventional cylindrical bore triode. These three types of accelerators are: (a) a cylindrical bore tetrode, (b) a triode with various shapes of the plasma electrode, and (c) a triode with the plasma electrode at elevated potential from the source plasma maintained by use of an insulator. All of these designs reduce the residual aberration fields from that of the cylindrical bore triode for reasons which are different for each type. We will explain these reasons as well as the virtues and potential difficulties with each of these types, by reference to the actual transmission of ions compared to those available, the structure of the ion beam velocity distribution function and stability with respect to source plasma fluctuations.

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\*Research sponsored by the Energy Research and Development Administration under contract with the Union Carbide Corporation.

## Ambipolar Potential of Tandem Mirrors\*

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The tandem mirror fusion reactor concept of Fowler and Logan<sup>1</sup> utilizes the positive space charge of mirror confined plasmas to plug the ends of a simple solenoid, which confines the fusion plasma. The self-consistent space charge potential distribution in such a system is therefore of interest. The condition that the Debye Length  $\lambda_D \ll$  system characteristic length  $L \ll$  Coulomb scattering mean-free-path  $\ell$ , permits the potential distribution in each region to be calculated if the confined particle velocity distribution functions are known<sup>2</sup>. In the tandem mirror system, the potential distribution is distorted such that there exists both locally confined particles in each of the end plug and central solenoid regions, and transient particles which are common to both regions. These transient particles serve to couple the two regions. The self-consistent ambipolar potential distribution is calculated by balancing the density of the transient particles in the two regions, which is determined by the particle orbits constrained by the constants of motion. The density ratio between the two regions is shown to be expressible as the Boltzmann well distribution corrected by a factor due to the effect of the inhomogeneous magnetic field. The dependence of the potential on the magnetic field configuration also determines the penetration of the ions confined in the central solenoid region into the end plugs which is expected to enhance the microstability of the system; and the penetration of wall emitted cold secondary electrons which is expected to degrade the energy confinement of the system.

\*Supported by ERDA under contract E(11-1)-3070

1. T.K. Fowler and G.B. Logan, UCRL-78740 (Preprint submitted to comments on Plasma Physics and Controlled Fusion), Oct. 11, 1976.
2. B.J. BenDaniel, Plasma Physics 3, 235(1961).

POSTER SESSION J

# Abstract

## A Renormalized Dielectric Function For Collisionless Drift Wave Turbulence\*

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Earlier renormalized theories for collisionless drift wave turbulence were deficient in several respects when  $\omega > k_z v_i$  and the principal perpendicular motion is the  $\underline{E} \times \underline{B}$  drift. The earlier theories did not preserve the perpendicularity between the  $\underline{E} \times \underline{B}$  ion current and the electric field  $\underline{E}$  and consequently did not conserve energy. In addition, earlier theories did not correctly treat the case  $n_{ion} \approx e\phi/T$  in which the nonlinear convective term  $\underline{\nabla}\phi \times \underline{B} \cdot \underline{\nabla} n_{ion}$  should be very small. These problems are analyzed and corrected in a new version. The new theory predicts that drift wave energy is converted into parallel ion energy, producing a damping rate  $\gamma = (k_z v_i / \omega)^2 k_\perp^2 D$  where  $D$  is a test particle diffusion coefficient. For a fluctuating potential  $\phi$  due only to drift waves,  $D$  is greater than zero only when the threshold  $e\phi/T \approx (k_x L_n)^{-1}$  for ion stochastic motion is exceeded. The nonlinear stabilization will occur slightly above this threshold if  $\gamma^{(e)} / \omega < (k_z v_i / \omega)^2$  [ $\gamma^{(e)}$  is the linear growth rate]. For this case the density diffusion coefficient  $D_n$  (which is greater than  $D$ ) will be  $D_n \approx \gamma^{(e)} / k_x^2$ .

\*This work is supported by ERDA

Ripple Assisted Neutral Injection Into  
Present and Future Tokamaks

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We present the results of a series of calculations related to ripple assisted neutral injection<sup>1</sup> in a present-day tokamak and in a future High Field Compact Test Reactor.<sup>2</sup> We extend the criterion for super-banana trapping developed by Anderson and Furth<sup>3</sup> to the case of non-sinusoidally varying ripple fields. Using this criterion we find optimal "eye-glass" coil designs for inducing asymmetric toroidal field ripple. A formula is found for the ripple coil periodicity required to obtain penetration of the ripple fields to the magnetic axis. For the case with sinusoidal field ripple, we show Monte-Carlo calculations of beam penetration and slowing down.

In addition, we study the magnetic islands produced by realistic ripple injection coils. For a single injector, we find significant islands, but no sign of ergodicity. With such a coil system one could easily study the effects of magnetic islands on plasma confinement. For practical numbers of injectors (4-8), the perturbation of the magnetic structure is negligible.

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\* Work supported by U.S.E.R.D.A. Contract E(11-1)-3073.

<sup>1</sup> D. L. Jassby, R. J. Goldston, Nucl. Fus. 16, 613 (1976).

<sup>2</sup> D. R. Cohn, et al., 2nd ANS Meeting on Technology of Controlled Nuclear Fusion 931 (Richland, 1976).

<sup>3</sup> O. A. Anderson, H. P. Furth, Nucl. Fus. 12, 207 (1972).

TESLA'S BALL LIGHTNING THEORY, A BGK-WAVE SOLITON-PAIR LANGMUIR-DEBYE  
SHEATH SURFACE-TENSION HYPOTHESIS, THE FARNSWORTH EFFECT, AND THE  
"HYDROTRON" ELECTROSTATIC-INERTIAL SELF-CONFINED PLASMOID CONCEPT

by Robert K. Golka, Golka Associates, Wendover, Utah 84083

and Robert W. Bass, Eyring Research Institute, Provo, Utah 84601

In 1899, Nikola Tesla experimented in his laboratory at Colorado Springs with a Tesla Coil having a 50,000 watt input, a peak current of 1,100 amperes at 12,500,000 volts, and a breakwheel frequency of 4,200 pulses per second, yielding up to 12 joules per pulse. Theoretically, this is enough energy to produce a low density, high temperature air bubble of radius 1.8 cm (assuming 50% efficiency). Tesla noted in his private diary observations of fireballs 1.9 cm in radius. An attempt has been made to precisely replicate the original Tesla apparatus, and to develop a modern plasma-physics theory that would make predictions compatible with his observations. M. Uman has calculated that a reasonable contamination with carbon (from vegetation) or copper (from wires) would eliminate the hot air bubble's excess buoyancy. M. A. Cook has demonstrated by theory and experiment that the Saha equation requires modification and that ionization is significant (1%) at 2,000 °K and that the plasmoid exhibits completely self-confining volumetric cohesion due to Madelung forces (a dynamic lattice or fluid crystal) up to 2,500 °K. J. L. Tuck has emphasized that observations of natural ball lightning require surface tension since the kugelblitzen "bounce". Following a suggestion of D. Rose and M. Clark, the BFKK Energy Principle is modified to include a surface tension (as suggested by W. L. Newcomb) arising from hypothesized radial electric fields in a thin boundary layer, yielding a rigorous proof of global asymptotic hydrodynamic stability of a plasma equilibrium confined according to Laplace's Soap Bubble Formula. The boundary layer can be explained by hypothesizing trapped particle layers, or Bernstein-Greene-Kruskal (BGK) static waves, in a soliton-pair electrostatic potential configuration which can be proved globally stable by a method of Rosenbluth and Oberman (first integral of the Vlasov equation). (This theory may solve Langmuir's Paradox.) The dipole boundary contains what Farnsworth called a virtual anode and a virtual cathode in his design of a spherical fusion reactor. The Hydrotron configuration is similar, except that it involves a hot plasma self-confined inside a cold plasma, by the above theory.

Stationary Spectrum of Strong Turbulence in Magnetized Nonuniform Plasma. Akira Hasegawa and Kunioki Mima, Bell Laboratories, Murray Hill, NJ, 07974. -- A simple nonlinear equation is derived to describe quasi-two-dimensional electrostatic dynamics of a nonuniform magnetized plasma. The equation, in the absence of a density gradient, has a structure which resembles to the two-dimensional Navier-Stokes equation. The nonlinearity originates from the polarization drift of ions and it produces nonlinear correction of order unity even if  $e\phi/T_e \sim \omega^*/\omega_{ci} \ll 1$ . A stationary spectrum for strong electrostatic turbulence is obtained using the model equation and assuming a source in a long wavelength region. The  $\omega$ -integrated  $k$  spectrum is given by  $k^{1.8}(1+k^2)^{-2.2}$ , while the width of the frequency spectrum (centered at  $\omega \sim 0$ ) is proportional to  $k^3(1+k^2)^{-1}$ , where  $k$  is normalized to  $(T_e/m_i)^{1/2}/\omega_{ci}$ . The results compare well with the recently observed spectrum in the ATC Tokamak.

## Convective Loss Cone Instability Driven by Energetic Ion Beams \*

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## Abstract

One method of heating which is currently being experimented with and which is planned for future toroidal devices is injection with energetic neutral beams. It has been demonstrated that such beams can heat a plasma effectively. However, if the injector is not parallel to the magnetic field, regions of  $\partial F_b / \partial v_{\perp}^2 > 0$  can develop and drive the convective loss cone instability. We study this instability for parameters of interest for current and next generation tokamaks by a simple slab model including magnetic shear and density gradients and we find that the most dangerous modes are strongly shear localized but not strongly shear damped. Therefore, the linear theory yields low beam density thresholds and significant ( $\sim \omega_{ci}$ ) growth rates for this instability. We have also considered quasilinear effects on the beam distribution function to examine saturation of the instability and try to determine its effect on heating rates and the macroscopic stability of the plasma.

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# Drift Convective Cells in a Sheared Magnetic Field<sup>\*</sup>

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The convective cell, i.e.,  $\omega \approx -i\mu k_{\perp}^2$  ( $\mu$ , the viscosity coefficient) has been an important candidate for explaining anomalous diffusion in a homogenous 2-D magnetoplasma because of its long life (low damping rate) and essentially d.c. character. Rosenbluth and Liu showed that in a sheared magnetic field, the fluctuation energy in this mode will be small; thus limiting its importance. Okuda and Dawson, showed that collisions could allow sufficient fluctuation levels. It must be noted that the maximum allowable energy for this damped mode is the thermal fluctuation level,  $(1/2)k_B T$  per  $k$  mode.

However, in the presence of inhomogeneities, it is well known that growing drift modes are formed for large  $k_{\parallel}$ . A careful analysis of the local dispersion relation (electrostatic) reveals that naturally growing modes with  $\gamma \gg \omega_r$ , and  $\gamma \ll \omega^*$  are formed for finite but small  $k_{\parallel}$ . These modes are the finite  $k_{\parallel}$  extension of the convective cell ( $k_{\parallel} = 0$ ), and the small  $k_{\parallel}$  extension of the conventional drift waves.

These drift convective cells are studied in a sheared magnetic field, and we find that these are among growing normal modes of the system. However, they do not need sound support for their formation as the conventional drift waves do. Since the frequency of these modes is much smaller than  $\omega^*$ , the results can explain the recent experimental observations of low frequency growing modes in FM-1 when operated in the collisional regime. The importance of these naturally growing modes for anomalous diffusion is also discussed.

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THE VLASOV-POISSON EQUATIONS -- ANOTHER VIEW  
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Motivation: Ascertain the structure of the nonlinear equations governing plasma oscillation in space and time.

Elementary operations applied to the one-dimensional Vlasov-Poisson equations lead to a nonlinear Klein-Gordon equation. In the cold plasma limit, this form embodies the well-known results for nonlinear waves in cold plasma. Linearization leads automatically to Langmuir waves in warm plasma. Also, in this context, trapping time is shown explicitly to measure the time at which nonlinearities can no longer be neglected.

TWO-DIMENSIONAL LOCALIZATION OF NONLINEAR DRIFT WAVE  
AND FORMATION OF CONVECTIVE CELLS\*

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The two-dimensional self modulation of the finite amplitude, strongly dispersive (short wavelength) drift wave is explored. The quasilinear equations for the drift wave with amplitude inhomogeneity in both the azimuthal and radial directions are derived and used to investigate the modulational instabilities. The threshold of the instability is found to be quite low since the strong nonlinearity comes from the nonlinear polarization drift. The amplitude modulation produces a zero frequency cross-field drift of ions which leads to charge separation. At one hundred percent modulation, the zero-frequency electric field due to the charge separation easily becomes comparable to the carrier wavefield. The convective motion is induced by the electric field and the density profile is flattened, stabilizing the drift wave instability. The saturation level due to such a stabilization mechanism is also presented. The formation of connective cells in the presence of drift waves has recently<sup>(1)</sup> been observed in the computer experiment by Cheng and Okuda, which lends credence to the above theory.

<sup>(1)</sup>Phys. Rev. Lett. 38, 708 (1977).

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## Tokamak Heating with Intense Pulsed Ion Beams

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This paper<sup>1</sup> examines the heating of tokamak plasmas with intense, pulsed ion beams. Four problems are considered, injection into the torus, cross-field propagation in the vacuum region of the tokamak, cross-field propagation in the plasma, and energy deposition in the plasma. Our calculations indicate that energy can be efficiently deposited in the center of the plasma by suitably choosing beam parameters.

One possible application of ion beam heating is to the flux-conserving high beta tokamak. In order for poloidal flux to remain frozen in, the plasma must be very rapidly heated to a high temperature. Intense ion beams probably offer one of the best means of rapidly heating the plasma. They can be used to initially heat the plasma, even if the ultimate heating mechanism relies on some other scheme.

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<sup>1</sup>E. Ott and W. Manheimer, submitted to Nucl. Fusion; also Tokamak Heating with Intense Pulsed Ion Beams, Naval Research Laboratory Memorandum Report 3394.

## STABILITY BOUNDARIES FOR LOSS-CONE MODES\*

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## ABSTRACT

The linear properties of loss-cone modes in mirror machines is determined by solving for the normal mode solution along the magnetic field. Marginally stable regions are bounded by DCLC and the negative energy wave. The latter boundary is a sensitive function of the various dissipation processes. Included in this study are electron Landau (damping)(through the outgoing waves), the finite- $\beta$   $\nabla B$  drift for ions and electrons and the distortion in the ion orbits due to their finite transit time. Standard techniques follow the solution of the second order wave equation which necessitates integrating through many oscillations of the wave to satisfy the outgoing wave boundary condition. To avoid this difficulty, we transform to the first order Riccati equation, which follows the phase of the wave, enabling us to examine the lowest eigenmode only (the most unstable). Other advantages of this method will be discussed.

Stability boundaries with and without stream stabilization are presented for the standard mirror and for the high density plug in the tandem mirror configuration. Stability windows in reactor regimes are also presented.

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# SOLITON IN THE DAMPED NONLINEAR SCHROEDINGER EQUATION\*

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The nonlinear Schroedinger equation is a model for many wave systems when dispersion and nonlinearity are of the same order. Solitons, stable nonlinear wave solutions that keep their identity on collision, and an infinite set of conservation laws are basic features of this equation.

A detailed comparison of numerical soliton solutions of the nonlinear Schroedinger equation with damping is made to the analytical assumption that a soliton keeps its shape, while it damps as predicted by the first few conservation laws.

Different damping laws are investigated, all of the form  $\gamma_k = \epsilon |k|^b$ . A standing damped soliton with damping exponent  $b = 1$  develops small tails, but a soliton with damping exponent  $b \geq 2$  remains localized. For damping exponent  $b \neq 2$  the soliton form changes slightly, but this change has a sizable influence on the damping coefficient. In contrast, when  $b = 2$  the soliton shape and the damping coefficient are constant in time. This qualitative difference between  $b \neq 2$  and  $b = 2$  is explained using the scaling properties of the terms in the nonlinear Schroedinger equation.

The velocity of a soliton for  $b = 2$  decreases in agreement with the second conservation law. A qualitative explanation of this decrease, valid for  $b \geq 0$ ), is given.

For damping exponent  $b = 2$  a formula for the stationary shape of the damped soliton, correct to second order in  $\epsilon$ , is found. The damping rate is calculated to this order. This damping rate shows excellent agreement with the numerical solution.

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## Drift Wave Coupling via Stimulated Compton Scattering

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Density gradient driven collisionless drift waves are considered in a slab geometry, with  $B$  in the  $z$ ,  $\nabla N$  in the  $x$  direction. The linearly fastest growing electrostatic modes propagate in the  $y$ - $z$  plane. It is shown that these modes couple to slower growing modes with a component in the  $x$  direction via scattering from quasi modes in the  $x$ - $z$  plane. This process leads to a nearly isotropic drift wave spectrum in the  $x$ - $y$  plane at the early stages of the nonlinear evolution of the drift waves. This is consistent with recent  $CO_2$  laser scattering results<sup>1</sup>.

<sup>1</sup> C. M. Surko and R. E. Slusher, Phys. Rev. Lett. 37, 1747 (1976).

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## Nonlinear Motion of High Beta Tokamaks\*

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## Abstract

Reduced MHD equations which describe large aspect ratio, high beta tokamaks, have been derived. The equations are almost the same as in the low beta case, but include pressure terms. To lowest order in the inverse aspect ratio  $\epsilon$ , the plasma can be considered as a linear low beta theta pinch, satisfying the equilibrium condition  $p + \frac{1}{2} B_z^2 = \text{constant}$ , where  $p$  is the pressure and  $B_z$  is the longitudinal magnetic field. The equilibrium is stable unless  $\nabla \cdot \mathbf{v} = 0$ , where  $\mathbf{v}$  is the fluid velocity. By requiring incompressible flow, the fast magnetosonic time scale is eliminated. Motion on the slower kink mode time scale arises in the next order in  $\epsilon$ , driven by the longitudinal current and toroidal effects involving the pressure. The motion self-consistently maintains the theta pinch equilibrium, if the appropriate correction to  $\nabla \cdot \mathbf{v}$  is made. This choice of  $\nabla \cdot \mathbf{v}$  is also obtained by minimizing  $\delta W$ , the energy principle.

Numerical calculations have been performed using these equations. Equilibria are readily calculated by imposing axisymmetry and introducing a friction term to remove kinetic energy. Stability calculations indicate that the equilibria are unstable for very low values of beta. Nonlinear, three dimensional calculations have been performed, using the unstable equilibria as initial conditions. These results show that the linear modes can grow exponentially to very large amplitude and carry the plasma to the wall.

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THE ALFVEN-ION CYCLOTRON INSTABILITY\*

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The physical mechanism of the Alfvén-ion cyclotron instability due to ion temperature anisotropy in a high  $\beta$  plasma is explored. We have observed evidence for the proposed mechanism in runs on a magnetostatic particle simulation code. The mechanism involves the induced perpendicular current which is created by rotating a tilted anisotropic distribution function. The instability can be stabilized by mixing up the phase of the distribution rotation. Effects of longitudinal nonuniformity on this is studied: The lower  $k$  modes are stabilized, while the higher  $k$  modes are weakly stabilized; however, the high  $k$  modes saturated very quickly by quasilinear flattening. The transverse nonuniformity seems also very important. Finite Larmor radius effect can be of the same order of magnitude as the destabilizing term in a typical mirror machine configuration.

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A mathematical criterion for stochasticity  
with application to ion heating

by Yvain M. Treve

If the solutions of the differential equation  $\dot{x} = Ax + \epsilon f(t, x)$  are periodic and Lyapunov stable for  $\epsilon$  sufficiently small, it may happen that some of them become unstable for  $\epsilon$  larger than some critical value  $\epsilon_c$ . The points corresponding to these unstable solutions on a surface of section then fill an area in a seemingly random fashion instead of falling on closed curves. An approximate value of  $\epsilon_c$  can be computed for a large class of differential systems using the perturbation methods due to Cesari, Hale, and others. The results of Fukuyama et al. (Phys. Rev. Lett., 38, 28 March 1977) on ion heating by an electromagnetic wave propagating perpendicularly to a uniform magnetic field are discussed in the above context. This approach to the problem of stochasticity can be applied to more complicated problems like, for instance, ion heating when the wave vector has an arbitrary direction; it may also prove useful in the study of the behavior of the magnetic lines in toroidal configurations.

# What is the Critical $\beta$ for Ballooning Mode?

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It is well known that, if the magnetic field lines are tied to the conducting wall, the interchange instability will not appear below the critical  $\beta$ ,  $\beta_c$ , where  $\beta_c$  is

$$\beta_c \approx \frac{aR_c}{L^2},$$

where  $a$  is the density gradient length,  $R_c$  is the (bad) curvature, and  $L$  is the connection length. In toroidal geometry  $L \approx qR$  and  $R_c \approx R$ , where  $R$  is the major radius, if the good curvature section acts equivalent to the conducting wall. Then,  $\beta_c \approx (a/R)(1/q^2)$ .

However, Coppi, Rosenbluth, and others demonstrated in the 1960's, in the minimum average system, that the stabilizing effect of the good curvature section is not that effective. A simple extension of their theories for the finite  $\beta$  ballooning mode shows that the  $\beta_c$  should be given as

$$\beta_c \approx \frac{aR_c}{L^2} \frac{2h}{(1 - 2h^2)},$$

where  $h$  is the so-called well depth. In toroidal geometry,  $h \approx (a/R)$ ; thus,  $\beta_c \approx 2\varepsilon^2/q^2(1-2\varepsilon^2)$ . We therefore must conclude that without shear stabilization  $\beta_c$  is rather small. The effect of shear will be included, if the author could do the calculation before the meeting.